

## A GENERALIZATION OF THE ESSENTIAL GRAPH FOR MODULES OVER COMMUTATIVE RINGS

F. Soheilnia, Sh. Payrovi and A. Behtoei

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**ABSTRACT.** Let  $R$  be a commutative ring with nonzero identity and let  $M$  be a unitary  $R$ -module. The essential graph of  $M$ , denoted by  $EG(M)$  is a simple undirected graph whose vertex set is  $Z(M) \setminus \text{Ann}_R(M)$  and two distinct vertices  $x$  and  $y$  are adjacent if and only if  $\text{Ann}_M(xy)$  is an essential submodule of  $M$ . Let  $r(\text{Ann}_R(M)) \neq \text{Ann}_R(M)$ . It is shown that  $EG(M)$  is a connected graph with  $\text{diam}(EG(M)) \leq 2$ . Whenever  $M$  is Noetherian, it is shown that  $EG(M)$  is a complete graph if and only if either  $Z(M) = r(\text{Ann}_R(M))$  or  $EG(M) = K_2$  and  $\text{diam}(EG(M)) = 2$  if and only if there are  $x, y \in Z(M) \setminus \text{Ann}_R(M)$  and  $\mathfrak{p} \in \text{Ass}_R(M)$  such that  $xy \notin \mathfrak{p}$ . Moreover, it is proved that  $\text{gr}(EG(M)) \in \{3, \infty\}$ . Furthermore, for a Noetherian module  $M$  with  $r(\text{Ann}_R(M)) = \text{Ann}_R(M)$  it is proved that  $|\text{Ass}_R(M)| = 2$  if and only if  $EG(M)$  is a complete bipartite graph that is not a star.

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**F. Soheilnia, Sh. Payrovi** (Corresponding Author), and **A. Behtoei**

Department of Mathematics

Imam Khomeini International University

P. O. Box: 3414916818, Qazvin, Iran

e-mails: f.soheilnia@edu.ikiu.ac.ir (F. Soheilnia)

shpayrovi@sci.ikiu.ac.ir (Sh. Payrovi)

a.behtoei@sci.ikiu.ac.ir (A. Behtoei)