

MODULES FOR WHICH EVERY ENDOMORPHISM HAS A NON-TRIVIAL INVARIANT SUBMODULE

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ABSTRACT. All rings are commutative. Let M be a module. We introduce the property **(P)**: Every endomorphism of M has a non-trivial invariant submodule. We determine the structure of all vector spaces having **(P)** over any field and all semisimple modules satisfying **(P)** over any ring. Also, we provide a structure theorem for abelian groups having this property. We conclude the paper by characterizing the class of rings for which every module satisfies **(P)** as that of the rings R for which R/\mathfrak{m} is an algebraically closed field for every maximal ideal \mathfrak{m} of R .

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