

## ON $(M,N)$ -CLOSED IDEALS IN AMALGAMATED ALGEBRA

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Received: 18 February 2020; Revised: 3 June 2020; Accepted: 7 June 2020

Communicated by Abdullah Harmanci

**ABSTRACT.** Let  $R$  be a commutative ring with  $1 \neq 0$  and let  $m$  and  $n$  be integers with  $1 \leq n < m$ . A proper ideal  $I$  of  $R$  is called an  $(m, n)$ -closed ideal of  $R$  if whenever  $a^m \in I$  for some  $a \in R$  implies  $a^n \in I$ . Let  $f : A \rightarrow B$  be a ring homomorphism and let  $J$  be an ideal of  $B$ . This paper investigates the concept of  $(m, n)$ -closed ideals in the amalgamation of  $A$  with  $B$  along  $J$  with respect  $f$  denoted by  $A \bowtie_f J$ . Namely, Section 2 investigates this notion to some extensions of ideals of  $A$  to  $A \bowtie_f J$ . Section 3 features the main result, which examines when each proper ideal of  $A \bowtie_f J$  is an  $(m, n)$ -closed ideal. This allows us to give necessary and sufficient conditions for the amalgamation to inherit the radical ideal property with applications on the transfer of von Neumann regular,  $\pi$ -regular and semisimple properties.

**Mathematics Subject Classification (2020):** 13F05, 13A15, 13E05, 13F20, 13C10, 13C11, 13F30, 13D05

**Keywords:**  $(m, n)$ -Closed ideal, radical ideal, semi- $n$ -absorbing ideal, amalgamated algebra, von Neumann regular ring,  $\pi$ -regular ring, semisimple ring

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