

DERIVING SOME PROPERTIES OF STANLEY-REISNER RINGS FROM THEIR SQUAREFREE ZERO-DIVISOR GRAPHS

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ABSTRACT. Let Δ be a simplicial complex, I_Δ its Stanley-Reisner ideal and $R = K[\Delta]$ its Stanley-Reisner ring over a field K . In 2018, the author introduced the squarefree zero-divisor graph of R , denoted by $\Gamma_{\text{sf}}(R)$, and proved that if Δ and Δ' are two simplicial complexes, then the graphs $\Gamma_{\text{sf}}(K[\Delta])$ and $\Gamma_{\text{sf}}(K[\Delta'])$ are isomorphic if and only if the rings $K[\Delta]$ and $K[\Delta']$ are isomorphic. Here we derive some algebraic properties of R using combinatorial properties of $\Gamma_{\text{sf}}(R)$. In particular, we state combinatorial conditions on $\Gamma_{\text{sf}}(R)$ which are necessary or sufficient for R to be Cohen-Macaulay. Moreover, we investigate when $\Gamma_{\text{sf}}(R)$ is in some well-known classes of graphs and show that in these cases, I_Δ has a linear resolution or is componentwise linear. Also we study the diameter and girth of $\Gamma_{\text{sf}}(R)$ and their algebraic interpretations.

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