International Electronic Journal of Algebra
Volume 14 (2013) 1-9

SOME CHARACTERIZATIONS OF HOM-LEIBNIZ ALGEBRAS

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Received: 14 September 2011; Revised: 31 January 2013
Communicated by Sergei Silvestrov

Abstract. Some basic properties of Hom-Leibniz algebras are found. These properties are the Hom-analogue of corresponding well-known properties of Leibniz algebras. Considering the Hom-Akivis algebra associated to a given Hom-Leibniz algebra, it is observed that the Hom-Akivis identity leads to an additional property of Hom-Leibniz algebras, which in turn gives a necessary and sufficient condition for Hom-Lie admissibility of Hom-Leibniz algebras. A necessary and sufficient condition for Hom-power associativity of Hom-Leibniz algebras is also found.

Mathematics Subject Classification (2010):17A30, 17A20, 17A32, 17D99
Keywords: Hom-Akivis algebra, Hom-Leibniz algebra, Hom-power associativity

1. Introduction

The theory of Hom-algebras originated from the introduction of the notion of a Hom-Lie algebra by J.T. Hartwig, D. Larsson and S.D. Silvestrov [6] in the study of algebraic structures describing some q-deformations of the Witt and the Virasoro algebras. A Hom-Lie algebra is characterized by a Jacobi-like identity (called the Hom-Jacobi identity) which is seen as the Jacobi identity twisted by an endomorphism of a given algebra. Thus, the class of Hom-Lie algebras contains the one of Lie algebras.

Generalizing the well-known construction of Lie algebras from associative algebras, the notion of a Hom-associative algebra is introduced by A. Makhlouf and S.D. Silvestrov [13] (in fact the commutator algebra of a Hom-associative algebra is a Hom-Lie algebra). The other class of Hom-algebras closely related to Hom-Lie algebras is the one of Hom-Leibniz algebras [13] (see also [9]) which are the Hom-analogue of Leibniz algebras [10]. Extending the Loday's construction ([10]) of Leibniz algebras from dialgebras, D. Yau [14] introduced Hom-dialgebras and proved that every Hom-dialgebra gives rise to a Hom-Leibniz algebra. Roughly, a Hom-type generalization of a given type of algebras is defined by a twisting of the defining identities with a linear self-map of the given algebra. For various Hom-type algebras one may refer, e.g., to [3,8,11,12,16,17]. In [15] D. Yau showed a way of constructing Hom-type algebras starting from their corresponding untwisted algebras and a self-map.

In [10] (see also [4,5]) the basic properties of Leibniz algebras are given. The main purpose of this note is to point out that the Hom-analogue of some of these properties holds in Hom-Leibniz algebras (section 3). Considering the Hom-Akivis algebra associated to a given Hom-Leibniz algebra, we observe that the property in Proposition 3.3 is the expression of the Hom-Akivis identity. As a consequence we found a necessary and sufficient condition for the Hom-Lie admissibility of Hom-Leibniz algebras (Corollary 3.5). Generalizing power-associativity of rings and algebras [2], the notion of the (right) nth Hom-power x^n of an element x in a Hom-algebra is introduced by D. Yau [18], as well as Hom-power associativity of Hom-algebras. We found that $x^n = 0$, $n \geq 3$, for any x in a left Hom-Leibniz algebra $(L, \cdot, \alpha)$ and that $(L, \cdot, \alpha)$ is Hom-power associative if and only if $\alpha(x)x^2 = 0$, for all x in L (Theorem 3.8). Then we deduce , as a particular case, corresponding characterizations of left Leibniz algebras (Corollary 3.9). Apart of the (right) nth Hom-power of an element of a Hom-algebra [18], we consider in this note the left nth Hom-power of the given element. This allows to prove the Hom-analogue (see Theorem 3.11) of a result of D.W. Barnes ([5], Theorem 1.2 and Corollary 1.3) characterizing left Leibniz algebras. In section 2 we recall some basic notions on Hom-algebras. Modules, algebras, and linearity are meant over a ground field $\mathbb{K}$ of characteristic 0.

2. Preliminaries

In this section we recall some basic notions related to Hom-algebras. These notions are introduced in [6,8,11,13,15].

Definition 2.1. A *Hom-algebra* is a triple $(A, \cdot, \alpha)$ in which A is a $\mathbb{K}$-vector space, " $\cdot$ " a binary operation on A and $\alpha : A \to A$ is a linear map (the twisting map) such that $\alpha(x \cdot y) = \alpha(x) \cdot \alpha(y)$ (multiplicativity), for all x, y in A.

Remark 2.2. *A more general notion of a Hom-algebra is given (see, e.g., [11], [13]) without the assumption of multiplicativity and A is considered just as a $\mathbb{K}$-module. For convenience, here we assume that a Hom-algebra $(A, \cdot, \alpha)$ is always multiplicative and that A is a $\mathbb{K}$-vector space.*

Definition 2.3. Let $(A, \cdot, \alpha)$ be a Hom-algebra.

(i) The *Hom-associator* of $(A, \cdot, \alpha)$ is the trilinear map $as : A \times A \times A \to A$ defined by $as(x, y, z) = (x \cdot y) \cdot \alpha(z) - \alpha(x) \cdot (y \cdot z)$, for all x, y, z in A.

(ii) $(A, \cdot, \alpha)$ is said to be *Hom-associative* if $as(x, y, z) = 0$ (Hom-associativity), for all x, y, z in A.

Remark 2.4. *If $\alpha = Id$ (the identity map) in $(A, \cdot, \alpha)$, then its Hom-associator is just the usual associator of the algebra $(A, \cdot)$. In Definition 2.1, the Hom-associativity is not assumed, i.e. $as(x, y, z) \neq 0$ in general. In this case $(A, \cdot, \alpha)$ is said non-Hom-associative [8] (or Hom-nonassociative [15]; in [12], $(A, \cdot, \alpha)$ is also called a nonassociative Hom-algebra). This matches the generalization of associative algebras by the nonassociative ones.*

Definition 2.5. (i) A *(left) Hom-Leibniz algebra* is a Hom-algebra $(A, \cdot, \alpha)$ such that the identity

$$\alpha(x) \cdot (y \cdot z) = (x \cdot y) \cdot \alpha(z) + \alpha(y) \cdot (x \cdot z) \tag{2.1}$$

holds for all x, y, z in A.

(ii) A *Hom-Lie algebra* is a Hom-algebra $(A, [-, -], \alpha)$ such that the binary operation "$[-, -]$" is skew-symmetric and the *Hom-Jacobi identity*

$$J_\alpha(x, y, z) = 0 \tag{2.2}$$

holds for all x, y, z in A, and $J_\alpha(x, y, z) := [[x, y], \alpha(z)] + [[y, z], \alpha(x)] + [[z, x], \alpha(y)]$ is called the *Hom-Jacobian.*

Remark 2.6. *The original definition of a Hom-Leibniz algebra [13] is related to the identity*

$$(x \cdot y) \cdot \alpha(z) = (x \cdot z) \cdot \alpha(y) + \alpha(x) \cdot (y \cdot z) \tag{2.3}$$

which is expressed in terms of (right) adjoint homomorphisms $Ad_y x := x \cdot y$ of $(A, \cdot, \alpha)$. This justifies the term of "(right) Hom-Leibniz algebra" that could be used for the Hom-Leibniz algebra defined in [13]. The dual of (2.3) is (2.1) and in this note we consider only left Hom-Leibniz algebras. For $\alpha = Id$ in $(A, \cdot, \alpha)$ (resp. $(A, [-, -], \alpha)$), any Hom-Leibniz algebra (resp. Hom-Lie algebra) is a Leibniz algebra $(A, \cdot)$ [4], [10] (resp. a Lie algebra $(A, [-, -])$). As for Leibniz algebras, if the operation "$\cdot$" of a given Hom-Leibniz algebra $(A, \cdot, \alpha)$ is skew-symmetric, then $(A, \cdot, \alpha)$ is a Hom-Lie algebra (see [13]).

In terms of Hom-associators, the identity (2.1) is written as

$$as(x, y, z) = -\alpha(y) \cdot (x \cdot z) \tag{2.4}$$

Therefore, from Definition 2.3 and Remark 2.4, we see that Hom-Leibniz algebras are examples of non-Hom-associative algebras.

Definition 2.7. [8] A *Hom-Akivis algebra* is a quadruple $(A, [-, -], [-, -, -], \alpha)$ in which A is a vector space, "$[-, -]$" a skew-symmetric binary operation on A, "$[-, -, -]$" a ternary operation on A and $\alpha : A \to A$ a linear map such that the *Hom-Akivis identity*

$$J_\alpha(x, y, z) = \circlearrowleft_{(x,y,z)} [x, y, z] - \circlearrowleft_{(x,y,z)} [y, x, z] \tag{2.5}$$

holds for all x, y, z in A, where $\circlearrowleft_{(x,y,z)}$ denotes the sum over cyclic permutation of x, y, z.

Note that when $\alpha = Id$ in a Hom-Akivis algebra $(A, [-,-], [-,-,-], \alpha)$, then one gets an **Akivis algebra** $(A, [-,-], [-,-,-])$. Akivis algebras were introduced in [1] (see also references therein), where they were called W-algebras. The term "Akivis algebra" for these objects is introduced in [7].

In [8], it is observed that to each non-Hom-associative algebra is associated a Hom-Akivis algebra (this is the Hom-analogue of a similar relationship between nonassociative algebras and Akivis algebras [1]). In this note, we use the specific properties of the Hom-Akivis algebra associated to a given Hom-Leibniz algebra to derive a property characterizing Hom-Leibniz algebras.

3. Characterizations

In this section, Hom-versions of some well-known properties of left Leibniz algebras are displayed. Considering the specific properties of the binary and ternary operations of the Hom-Akivis algebra associated to a given Hom-Leibniz algebra, we infer a characteristic property of Hom-Leibniz algebras (Proposition 3.3). This property in turn allows to give a necessary and sufficient condition for the Hom-Lie admissibility of these Hom-algebras (Corollary 3.5). The Hom-power associativity of Hom-Leibniz algebras is considered.

Let $(A, \cdot, \alpha)$ be a Hom-Leibniz algebra and consider on $(A, \cdot, \alpha)$ the operations

$$[x, y] := x \cdot y - y \cdot x \tag{3.1}$$

$$[x, y, z] := as(x, y, z) \tag{3.2}$$

Then the operations (3.1) and (3.2) define on A a Hom-Akivis structure ([8]). We have the following proposition:

Proposition 3.1. *Let $(A, \cdot, \alpha)$ be a Hom-Leibniz algebra. Then*

(i) $(x \cdot y + y \cdot x) \cdot \alpha(z) = 0$,

(ii) $\alpha(x) \cdot [y, z] = [x \cdot y, \alpha(z)] + [\alpha(y), x \cdot z]$, *for all x, y, z in A.*

Proof. The identity (2.1) implies that $(x \cdot y) \cdot \alpha(z) = \alpha(x) \cdot (y \cdot z) - \alpha(y) \cdot (x \cdot z)$. Likewise, interchanging x and y, we have $(y \cdot x) \cdot \alpha(z) = \alpha(y) \cdot (x \cdot z) - \alpha(x) \cdot (y \cdot z)$. Then, adding memberwise these equalities above, we come to the property (i). Next we have

$$\begin{aligned} [x \cdot y, \alpha(z)] + [\alpha(y), x \cdot z] &= (x \cdot y) \cdot \alpha(z) - \alpha(z) \cdot (x \cdot y) \\ &+ \alpha(y) \cdot (x \cdot z) - (x \cdot z) \cdot \alpha(y) \\ &= \alpha(x) \cdot (y \cdot z) - \alpha(z) \cdot (x \cdot y) - (x \cdot z) \cdot \alpha(y) \text{ (by (2.1))} \\ &= \alpha(x) \cdot (y \cdot z) - (z \cdot x) \cdot \alpha(y) - \alpha(x) \cdot (z \cdot y) \\ &- (x \cdot z) \cdot \alpha(y) \text{ (by (2.1))} \\ &= \alpha(x) \cdot (y \cdot z) - \alpha(x) \cdot (z \cdot y) \text{ (by (i))} \\ &= \alpha(x) \cdot [y, z] \end{aligned}$$

and so we get (ii). □

Remark 3.2. *If set $\alpha = Id$ in Proposition 3.1, then one recovers the well-known properties of Leibniz algebras: $(x\cdot y + y\cdot x)\cdot z = 0$ and $x\cdot[y,z] = [x\cdot y, z] + [y, x\cdot z]$ (see [4], [10]).*

Proposition 3.3. *Let $(A,\cdot,\alpha)$ be a Hom-Leibniz algebra. Then*

$$J_\alpha(x,y,z) = \circlearrowleft_{(x,y,z)}(x\cdot y)\cdot\alpha(z), \tag{3.3}$$

for all x,y,z in A.

Proof. Considering (2.5) and then applying (3.2) and (2.4), we get $J_\alpha(x,y,z) = \circlearrowleft_{(x,y,z)}[-\alpha(y)\cdot(x\cdot z)] - \circlearrowleft_{(x,y,z)}[-\alpha(x)\cdot(y\cdot z)] = \circlearrowleft_{(x,y,z)}[\alpha(x)\cdot(y\cdot z) - \alpha(y)\cdot(x\cdot z)] = \circlearrowleft_{(x,y,z)}(x\cdot y)\cdot\alpha(z)$ (by (2.1)). □

One observes that (3.3) is the specific form of the Hom-Akivis identity (2.5) in case of Hom-Leibniz algebras.

Definition 3.4. [13] A Hom-algebra $(A,\cdot,\alpha)$ is said to be *Hom-Lie admissible* if $(A,[-,-],\alpha)$ is a Hom-Lie algebra, where $[x,y] := x\cdot y - y\cdot x$ for all x,y in A.

The skew-symmetry of the operation "$\cdot$" of a Hom-Leibniz algebra $(A,\cdot,\alpha)$ is a condition for $(A,\cdot,\alpha)$ to be a Hom-Lie algebra [13]. From Proposition 3.3 one gets the following necessary and sufficient condition for the Hom-Lie admissibility [13] of a given Hom-Leibniz algebra.

Corollary 3.5. *A Hom-Leibniz algebra $(A,\cdot,\alpha)$ is Hom-Lie admissible if and only if $\circlearrowleft_{(x,y,z)}(x\cdot y)\cdot\alpha(z) = 0$, for all x,y,z in A.*

In [18] D. Yau introduced Hom-power associative algebras which are seen as a generalization of power-associative algebras. It is shown that some important properties of power-associative algebras are reported to Hom-power associative algebras.

Let A be a Hom-Leibniz algebra with a twisting linear self-map α and the binary operation on A denoted by juxtaposition. We recall the following definition.

Definition 3.6. [18] Let $x \in A$ and denote by α^m the m-fold composition of m copies of α with $\alpha^0 := Id$.

(1) The *nth Hom-power* $x^n \in A$ of x is inductively defined by

$$x^1 = x, \quad x^n = x^{n-1}\alpha^{n-2}(x) \tag{3.4}$$

for $n \geq 2$.

(2) The Hom-algebra A is *nth Hom-power associative* if

$$x^n = \alpha^{n-i-1}(x^i)\alpha^{i-1}(x^{n-i}) \tag{3.5}$$

for all $x \in A$ and $i \in \{1,...,n-1\}$.

(3) The Hom-algebra A is *up to nth Hom-power associative* if A is kth Hom-power associative for all $k \in \{2,...,n\}$.

(4) The Hom-algebra A is *Hom-power associative* if A is nth Hom-power associative for all $n \geq 2$.

The following result provides a characterization of third Hom-power associativity of Hom-Leibniz algebras.

Lemma 3.7. *Let $(A, \cdot, \alpha)$ be a Hom-Leibniz algebra. Then*

(i) $x^3 = 0$*, for all* $x \in A$*;*

(ii) $(A, \cdot, \alpha)$ *is third Hom-power associative if and only if* $\alpha(x)x^2 = 0$*, for all* $x \in A$*.*

Proof. From (3.4) we have $x^3 := x^2\alpha(x)$. Therefore, the assertion (i) follows from Proposition 3.1(i) if set $y = x = z$. Next, from (3.5) we note that the $i = 2$ case of nth Hom-power associativity is automatically satisfied since this case is $x^3 = \alpha^0(x^2)\alpha^1(x^1) = x^2\alpha(x)$, which holds by definition. The $i = 2$ case says that $x^3 = \alpha^1(x)\alpha^0(x^2) = \alpha(x)x^2$. Therefore, since $x^2\alpha(x) = 0$ naturally holds by Proposition 3.1 (i), we conclude that the third Hom-power associativity of $(A, \cdot, \alpha)$ holds if and only if $\alpha(x)x^2 = 0$ for all $x \in A$, which proves the assertion (ii). □

The following result shows that the condition in Lemma 3.7 is also necessary and sufficient for the Hom-power associativity of $(A, \cdot, \alpha)$. To prove this, we rely on the main result of [18] (see Corollary 5.2).

Theorem 3.8. *Let $(A, \cdot, \alpha)$ be a Hom-Leibniz algebra. Then*

(i) $x^n = 0$*,* $n \geq 3$*, for all* $x \in A$*;*

(ii) $(A, \cdot, \alpha)$ *is Hom-power associative if and only if* $\alpha(x)x^2 = 0$*, for all* $x \in A$*.*

Proof. The proof of (i) is by induction on n: the first step $n = 3$ holds by Lemma 3.7(i); now if suppose that $x^n = 0$, then $x^{n+1} := x^{(n+1)-1}\alpha^{(n+1)-2}(x) = x^n\alpha^{n-1}(x) = 0$ so we get (i).

Corollary 5.2 of [18] says that, for a multiplicative Hom-algebra, the Hom-power associativity is equivalent to both of the conditions

$$x^2\alpha(x) = \alpha(x)x^2 \text{ and } x^4 = \alpha(x^2)\alpha(x^2). \tag{3.6}$$

In the situation of multiplicative left Hom-Leibniz algebras, the first equality of (3.6) is satisfied by Lemma 3.7(i) and the hypothesis $\alpha(x)x^2 = 0$. We have the following from (3.5):

Case $i = 1$: $x^4 := \alpha^{4-2}(x)\alpha^0(x^3) = \alpha^2(x)x^3$,

Case $i = 2$: $x^4 := \alpha(x^2)\alpha(x^2)$,

Case $i = 3$: $x^4 := \alpha^0(x^3)\alpha^2(x) = x^3\alpha^2(x)$.

Because of the assertion (i) above, only the case $i = 2$ is of interest here. From one side we have $x^4 = 0$ (by (i)) and, from the other side we have $\alpha(x^2)\alpha(x^2) =$

$[\alpha(x)]^2\alpha(x^2) = 0$ (by multiplicativity and Proposition 3.1(i)). Therefore, Corollary 5.2 of [18] now applies and we conclude that (3.6) holds (i.e. $(A, \cdot, \alpha)$ is Hom-power associative) if and only if $\alpha(x)x^2 = 0$, which proves (ii). □

Let A be an algebra (over a field of characteristic 0). For an element $x \in A$, the **right powers** are defined by

$$x^1 = x, \text{ and } x^{n+1} = x^n x \tag{3.7}$$

for $n \geq 1$. Then A is power-associative if and only if

$$x^n = x^{n-i}x^i \tag{3.8}$$

for all $x \in A$, $n \geq 2$, and $i \in \{1, ..., n-1\}$. By a theorem of Albert [2] , A is power-associative if only if it is third and fourth power-associative, which in turn is equivalent to

$$x^2x = xx^2 \text{ and } x^4 = x^2x^2. \tag{3.9}$$

for all $x \in A$.

Some consequences of the results above are the following simple characterizations of (left) Leibniz algebras.

Corollary 3.9. *Let $(A, \cdot)$ be a left Leibniz algebra. Then*

(i) *$x^n = 0$, $n \geq 3$, for all $x \in A$;*
(ii) *$(A, \cdot)$ is power-associative if and only if $xx^2 = 0$, for all $x \in A$.*

Proof. The part (i) of this corollary follows from (3.7) and Theorem 3.8(i) when $\alpha = Id$ (we used here the well-known property $(xy + yx)z = 0$ of left Leibniz algebras). The assertion (ii) is a special case of Theorem 3.8(ii) (when $\alpha = Id$), if keep in mind the assertion (i), (3.8), and (3.9). □

Remark 3.10. *Although the condition $xx^2 = 0$ does not always hold in a left Leibniz algebra $(A, \cdot)$, we do have $xx^2 \cdot z = 0$ for all $x, z \in A$ (again, this follows from the property $(xy + yx)z = 0$). In fact, $b \cdot z = 0$, $z \in A$, where $b \neq 0$ is a left mth power of x ($m \geq 2$), i.e. $b = x(x(...(xx)...))$ ([5], Theorem 1.2 and Corollary 1.3).*

Let us call the *nth right Hom-power* of $x \in A$ the power defined by (3.4), where A is a Hom-algebra. Then one may consider the *nth left Hom-power* of $a \in A$ defined by

$$a^1 = a, \quad a^n = \alpha^{n-2}(a)a^{n-1} \tag{3.10}$$

for $n \geq 2$. In this setting of left Hom-powers, we have the following theorem.

Theorem 3.11. *Let $(A, \cdot, \alpha)$ be a Hom-Leibniz algebra and let $a \in A$. Then $L_{a^n} \circ \alpha = 0$, $n \geq 2$, where L_z denotes the left multiplication by z in $(A, \cdot, \alpha)$, i.e. $L_z x = z \cdot x$, $x \in L$.*

Proof. We proceed by induction on n and the repeated use of Proposition 3.1(i). From Proposition 3.1(i), we get $a^2\alpha(z) = 0$, $\forall a, z \in A$ and thus the first step $n = 2$

is verified. Now assume that, up to the degree n, we have $a^n\alpha(z) = 0$, $\forall a, z \in A$. Then Proposition 3.1(i) implies that $(a^n\alpha^{n-1}(a) + \alpha^{n-1}(a)a^n)\alpha(z) = 0$, i.e. $(a^n\alpha(\alpha^{n-2}(a)) + \alpha^{n-1}(a)a^n)\alpha(z) = 0$. The application of the induction hypothesis to $a^n\alpha(\alpha^{n-2}(a))$ leads to $(\alpha^{n-1}(a)a^n)\alpha(z) = 0$, i.e. $(\alpha^{(n+1)-2}(a)a^{(n+1)-1})\alpha(z) = 0$ which means (by (3.10)) that $a^{n+1}\alpha(z) = 0$. Therefore, we conclude that $a^n\alpha(z) = 0$, $\forall n \geq 2$, i.e. $L_{a^n} \circ \alpha = 0$, $n \geq 2$. □

Remark 3.12. *We observe that Theorem 3.11 above is an α-twisted version of a result of D.W. Barnes ([5], Theorem 1.2 and Corollary 1.3), related to left Leibniz algebras. Indeed, setting $\alpha = Id$ in Theorem 3.11 we get the result of Barnes.*

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