International Electronic Journal of Algebra
Volume 14 (2013) 10-18

ON STRONGLY REGULAR RINGS AND GENERALIZATIONS OF V-RINGS

Tikaram Subedi and Ardeline Mary Buhphang

Received: 3 April 2012; Revised: 4 June 2013
Communicated by Dolors Herbera

Abstract. We prove that a left GP-V-ring is right non-singular. We also give some properties of left GP-V'-rings. Some characterizations of strongly regular rings and biregular rings are also given.

Mathematics Subject Classification (2010):16E50, 16D25, 16D40
Keywords: Von Neumann regular rings, strongly regular rings, GP-V-ring, GP-V'-ring, W-ideals

1. Introduction

Throughout this paper, R denotes an associative ring with identity and all our modules are unitary. The symbols $J(R)$, $Z(_RR)$ $(Z(R_R))$, $I(R)$ respectively stand for the Jacobson radical, the left (right) singular ideal and the set of all idempotent elements of R. R is *semiprimitive* if $J(R) = 0$. R is *left (right) non-singular* if $Z(_RR) = 0$ $(Z(R_R) = 0)$. For any subset X of R, $l(X)$ $(r(X))$ denotes the left (right) annihilator of X and for any $a \in R$, $l(a)$ denotes $l(\{a\})$ and $r(a)$ denotes $r(\{a\})$. By an *ideal*, we mean a two sided ideal. Following [1], a ring R is called *complement left (right) bounded* if every non-zero complement left (right) ideal of R contain a non-zero ideal of R. R is *(von Neumann) regular* if for every $a \in R$, there exists some $b \in R$ such that $a = aba$ and R is *strongly regular* if for every $a \in R$, there exists some $b \in R$ such that $a = a^2b$. Over the last few decades, regular rings and strongly regular rings are extensively studied (for example cf. [1]-[12]). Clearly, R is strongly regular if and only if R is a reduced regular ring. Following [9], R is *biregular* if for every $a \in R$, RaR is generated by a central idempotent; a left (right) ideal I of R is *reduced* if it contains no non-zero nilpotent elements. A subset A of R is *regular* if for every $a \in A$, there exists some $b \in R$ such that $a = aba$ ([2]). Also following [3], a left (right) R-module M is *YJ-injective* if for each $0 \neq a \in R$, there exists a positive integer n such that $a^n \neq 0$ and every left (right) R-homomorphism from Ra^n (a^nR) to M extends to a left (right) R-homomorphism from R to M; R is a *left (right) GP-V-ring* if every simple left (right) R-module is YJ-injective; R is a *left (right) GP-V'-ring* if every simple singular left (right) R-module is YJ-injective. By [7, Lemma 2], a regular ring is a left (right) GP-V-ring and hence a left (right) GP-V'-ring. But GP-V-rings are not necessarily regular (cf. [3]). R is called a *ZI (zero insertive)-ring* ([3]), if for every a, $b \in R, ab = 0$ implies $aRb = 0$. It is well known that R is ZI if and only if for every $a \in R$, $l(a)$ is an ideal of R if and only if $r(a)$ is an ideal of R. Following [12], a ring R is called a *left (right) SF-ring* if every simple left (right) R-module is flat. It is well known that a ring R is regular if and only if every left (right) R-module is flat, hence a regular ring is a left (right) SF-ring, but till date it is unknown whether SF-rings are necessarily regular.

This paper presents some new properties of GP-V-rings, GP-V'-rings and also proves the regularity of a class of SF-ring.

2. Main Results

Lemma 2.1. *For any $0 \neq a \in Z(R_R)$, aR contains a non-zero nilpotent element.*

Proof. Since $0 \neq a \in Z(R_R)$, $r(a) \cap RaR \neq 0$. Let $0 \neq x \in r(a) \cap RaR$. Then $x = rar$ for some $0 \neq r \in R$. Then $ar \neq 0$ and $(ar)^2 = arar = 0$. This proves the lemma. □

Theorem 2.2. *If R is a left GP-V-ring, then R is right non-singular.*

Proof. Suppose $Z(R_R) \neq 0$. Then by Lemma 2.1, there exists some $0 \neq a \in Z(R_R)$ such that $a^2 = 0$. If $l(a) + RaR \neq R$, let M be a maximal left ideal of R such that $l(a) + RaR \subseteq M$. Since R is a left GP-V-ring, R/M is YJ-injective. Since $a^2 = 0$, every left R-homomorphism from Ra to R/M extends to one from R to R/M. Define $f : Ra \longrightarrow R/M$ by $f(ra) = r + M$. Clearly f is well-defined. It follows that there exists some $b \in R$ such that $1 - ab \in M$. But $ab \in RaR \subseteq M$, whence $1 \in M$, a contradiction. Hence $l(a) + RaR = R$. This implies that $x + \sum y_i a z_i = 1$ for some $x \in l(a)$, $y_i, z_i \in R$. This yields $(1 - \sum y_i a z_i)a = 0$, that is $a \in r(1 - \sum y_i a z_i)$. Now $a \in Z(R_R)$, hence $r(\sum y_i a z_i)$ is essential right ideal of R. Therefore it follows from $r(1 - \sum y_i a z_i) \cap r(\sum y_i a z_i) = 0$ that $r(1 - \sum y_i a z_i) = 0$, whence $a = 0$. This is a contradiction to $a \neq 0$. Thus $Z(R_R) = 0$. □

Proposition 2.3. *If R is a left GP-V'-ring, then $Z(_RR) \cap Z(R_R) = 0$.*

Proof. Suppose $Z(_RR) \cap Z(R_R) \neq 0$, then there exists some $0 \neq a \in Z(_RR) \cap Z(R_R)$ such that $a^2 = 0$. If $l(a) + RaR \neq R$, let M be a maximal left ideal of R such that $l(a) + RaR \subseteq M$. Clearly M is essential so that R/M is YJ-injective. Proceeding as in Theorem 2.2, we get a contradiction. Thus $Z(_RR) \cap Z(R_R) = 0$. □

Proposition 2.4. *If R is a left GP-V'-ring, then the center C of R is a reduced ring.*

Proof. Let $0 \neq a \in C$ such that $a^2 = 0$. There exists a maximal left ideal M of R containing $l(a)$. If M is not essential, then $M = l(e)$ for some $0 \neq e \in I(R)$. Now $a \in l(a) \subseteq M = l(e)$, hence $ea = ae = 0$. Therefore $e \in l(a) \subseteq M = l(e)$ yielding $e = e^2 = 0$, a contradiction. Hence M is essential and so R/M is YJ-injective. It follows that there exists some $b \in R$ such that $1 - ab \in M$. But as $a \in C \cap M$, we have $ab \in M$, whence $1 \in M$, a contradiction. Therefore C is reduced. □

We recall the following two definitions following [12].

Definition 2.5. A left ideal L of a ring R is a *weak ideal (W-ideal)* if for all $0 \neq a \in L$, there exists a positive integer n such that $a^n \neq 0$ and $a^n R \subseteq L$. A right ideal K of R is defined similarly to be a W-ideal.

Definition 2.6. A left ideal L of a ring R is a *generalized weak ideal (GW-ideal)* if for all $a \in L$, there exists a positive integer n such that $a^n R \subseteq L$. A right ideal K of R is defined similarly to be a GW-ideal.

By ([12], Example 1.2), there exists a ring R in which { ideals of $R\} \subsetneq \{$ W-ideals of $R\} \subsetneq \{$ GW-ideals of $R\}$.

Lemma 2.7. *The following conditions are equivalent for a ring R.*

(1) *R is abelian.*
(2) *$l(e)$ is a GW-ideal of R for every $e \in I(R)$.*
(3) *$r(e)$ is a GW-ideal of R for every $e \in I(R)$.*

Proof. Clearly (1) ⇒ (2) and (1) ⇒ (3).
(2) ⇒ (1) Let $e \in I(R)$ and $x \in R$. Since $1 - e \in l(e)$ and $l(e)$ is a GW-ideal of R, there exists a positive integer n such that $(1 - e)^n x \in l(e)$ which implies that $xe = exe$. Again $1 - e \in I(R)$ and $e \in l(1 - e)$, arguing similarly we see that $ex = exe$. Hence, $ex = xe$. Thus, R is abelian. Similarly, we can prove (3) ⇒ (1). □

Theorem 2.8. *Let R be a left GP-V'-ring in which $l(e)$ is a GW-ideal of R for every $e \in I(R)$, then R is right non-singular.*

Proof. If $Z(R_R) \neq 0$, there exists some $0 \neq a \in Z(R_R)$ such that $a^2 = 0$. Suppose $l(a) + RaR \neq R$ and M be a maximal left ideal of R such that $l(a) + RaR \subseteq M$. If M is not essential, then $M = l(e)$ for for some $0 \neq e \in I(R)$. Since $a \in M = l(e)$, it follows from Lemma 2.7 that $ea = 0$. Hence $e \in l(a) \subseteq M = l(e)$, whence $e = e^2 = 0$, a contradiction to $e \neq 0$. Thus M is essential. Since R is a left GP-V'-ring, R/M is YJ-injective. Proceeding as in the proof of Theorem 2.2, we get a contradiction. Thus $l(a) + RaR = R$. Arguing again as in the proof of Theorem 2.2, we get a contradiction. This proves that $Z(R_R) = 0$. □

Corollary 2.9. *If R is a ZI, left GP-V'-ring, then R is right non-singular.*

Remark 2.10. *Corollary 2.9 shows that some of the hypotheses in* [3, Theorem 2.1] *are not necessary.*

Proposition 2.11. *The following conditions are equivalent.*

(1) *R is a complement left bounded ring.*
(2) *Every non-zero complement left ideal of R contains a non-zero left ideal which is a W-ideal.*

Proof. Clearly (1) $\Rightarrow$ (2).
(2) $\Rightarrow$ (1) Let L be a non-zero complement left ideal of R. By hypothesis, L contains a non-zero left ideal I of R which is a W-ideal. Let $0 \neq a \in I$. Then there exists some positive integer n such that $a^n \neq 0$ and $a^n R \subseteq I$. Therefore $Ra^n R \subseteq RI \subseteq RL = L$. Hence L contains the non-zero ideal $Ra^n R$. □

Proposition 2.12. *Let R be a complement left bounded ring. If $a \in R$ is nilpotent then $a \in Z({}_R R)$. In particular, a complement left bounded and left non singular ring is reduced.*

Proof. Let $0 \neq a \in R$ such that $a^2 = 0$. If $a \notin Z({}_R R)$ then every complement of $l(a)$ in R is non-zero. Let L be a complement of $l(a)$ in R. Then $l(a) \cap L = 0$ and $L \neq 0$. By hypothesis, L contains a non-zero ideal I of R . Let $0 \neq b \in I$, then

$$ba \in I \cap l(a) \subseteq L \cap l(a) = 0.$$

Therefore $b \in L \cap l(a) = 0$. This is a contradiction to $b \neq 0$. Hence $a \in Z({}_R R)$. This implies that a complement left bounded and a left non singular ring is reduced. □

Theorem 2.13. *Let R be a complement left bounded ring. The following conditions are equivalent.*

(1) *R is strongly regular.*
(2) *R is a left GP-V'-ring whose maximal essential left ideals are GW-ideals.*
(3) *R is a left GP-V'-ring whose maximal essential right ideals are GW-ideals.*

Proof. Clearly (1) $\Rightarrow$ (2) and (3).
(2) $\Rightarrow$ (1) Let $a \in R$. If $l(a) + Ra$ is not essential, then there exists a non-zero complement left ideal L of R such that $(l(a) + Ra) \cap L = 0$. Since R is complement left bounded, there exists a non-zero ideal I of R and $I \subseteq L$. Let $0 \neq x \in I$. Then

$$xa \in I \cap Ra = 0.$$

This implies that $x \in l(a) \cap I = 0$. This is a contradiction to $x \neq 0$. Therefore $l(a) + Ra$ is an essential left ideal of R. If $l(a) + Ra \neq R$, then there exists a maximal left ideal M of R such that $l(a) + Ra \subseteq M$. Since $l(a) + Ra$ is essential,

M is essential. Therefore it follows from R being a left GP-V'-ring that R/M is YJ-injective. Following the proof of (2) $\Rightarrow$ (1) of Theorem 3.1 of [6], we get a contradiction. Hence $l(a)+Ra=R$. Hence $x+ya=1$ for some $x \in l(a)$ and $y \in R$ which yields $a=ya^2$. This proves that R is strongly regular.
(3) $\Rightarrow$ (1) Suppose $0 \neq a \in R$ such that $a^2=0$. Let M be a maximal left ideal of R containing $l(a)$. If $a \notin Z(_RR)$, then proceeding as in Proposition 2.12, we get a contradiction. Hence $a \in Z(_RR)$ and so M is an essential left ideal of R. Since R is a left GP-V'-ring, it follows that there exists some $c \in R$ such that $1-ac \in M$. If $ac \notin M$, then $M+Rac=R$ implying $u+vac=1$ for some $u \in M$, $v \in R$. Since M is a GW-ideal and $cva \in M$, there exists $m>0$ such that $(cva)^m c \in M$. Therefore

$$(1-u)^{m+1} = (vac)^{m+1} = va(cva)^m c \in M.$$

This together with $u \in M$ implies that $1 \in M$, a contradiction. This proves that R is reduced and hence $l(w)=r(w)$ for all $w \in R$. Proceeding as in (3) $\Rightarrow$ (1) of [6, Theorem 3.1] we can prove that $l(a)+aR=R$ for every $a \in R$, which implies that R is regular. Since R is also reduced, R is strongly regular. □

Proposition 2.14. *Let R be a complement left bounded, left GP-V'-ring. Then*

(i) *R is semiprimitive.*
(ii) *R is right non-singular.*
(iii) *If a is a non-zero divisor of R, then $RaR=R$.*

Proof. (i). Suppose $0 \neq a \in J(R)$ such that $a^2=0$. If $a \notin Z(_RR)$, then there exists a non-zero complement left ideal L of R such that $l(a) \oplus L$ is an essential left ideal of R. Proceeding as in the proof of Proposition 2.12, we get a contradiction. Therefore $a \in Z(_RR)$. If $l(a)+RaR \neq R$, there exists a maximal essential left ideal M of R such that $l(a)+RaR \subseteq M$. Since R is a left GP-V'-ring, R/M is YJ-injective. It follows that there exists some $c \in R$ such that $1-ac \in M$. But $ac \in RaR \subseteq M$, whence $1 \in M$, a contradiction to $M \neq R$. Therefore $l(a)+RaR=R$. As $J(R)$ is small in R, $l(a)+RaR=R$ implies $l(a)=R$ so that $a=0$. This proves that $J(R)$ is reduced.

Suppose $0 \neq b \in J(R)$. If $l(b)+RbR$ is not essential, then there exists a non-zero complement left ideal K of R such that $(l(b)+RbR) \oplus K$ is an essential left ideal of R. By hypothesis, K contains a non-zero ideal I of R. Let $0 \neq d \in I$. Then

$$db \in K \cap RbR = 0,\ d \in K \cap l(b) = 0.$$

This is a contradiction to $d \neq 0$. Therefore $l(b)+RbR$ is essential. If $l(b)+RbR \neq R$, there exists a maximal left ideal L of R such that $l(b)+RbR \subseteq L$. Since $l(b)+RbR$ is essential. L is essential. Therefore R/L is YJ-injective. Arguing as in the proof of Theorem 2.2, we get a contradiction. Hence $l(b)+RbR=R$. Proceeding again as in the first paragraph of this proof we get $b=0$, a contradiction. This proves

that $J(R) = 0$.
(ii). Suppose $Z(R_R) \neq 0$. Then there exists some $0 \neq a \in Z(R_R)$ such that $a^2 = 0$. Suppose $l(a)+RaR \neq R$ and M be a maximal left ideal of R containing $l(a)+RaR$. If $l(a)+RaR$ is not essential, then there exists a non-zero complement left ideal K of R such that $(l(a) + RaR) \oplus K$ is essential left ideal of R. Since R is a complement left bounded ring, it is easy to see that $K = 0$. Hence $l(a) + RaR$ is essential and hence M is essential. Since R is a left GP-V'-ring, R/M is YJ-injective. Proceeding as in the proof of Theorem 2.2, we get a contradiction. Hence $l(a) + RaR = R$. Arguing again as in the proof of Theorem 2.2, we get a contradiction. Thus $Z(R_R) = 0$.
(iii). Suppose RaR is not an essential left ideal of R, then there exists a non-zero complement left ideal L of R such that $RaR \oplus L$ is an essential left ideal of R. By hypothesis L contains a non-zero ideal I of R. If $0 \neq b \in I$, then

$$ba \in I \cap RaR = 0.$$

This implies that $b \in l(a) = 0$, a contradiction to $b \neq 0$. Hence RaR is an essential left ideal of R. Suppose $RaR \neq R$ and M be a maximal left ideal of R containing RaR. Since RaR is essential and $RaR \subseteq M$, M is essential. Since R is a left GP-V'-ring, R/M is YJ-injective. Therefore there exists a positive integer n such that $a^n \neq 0$ and every left R- homomorphism from Ra^n to R/M extends to one from R to R/M. Define $f : Ra^n \longrightarrow R/M$ by $f(ra^n) = r + M$, for every $r \in R$. Since $l(a) = 0$, f is well-defined. It is clear that $1 - a^n b \in M$. Since $a^n b \in RaR \subseteq M$, it follows that $1 \in M$, a contradiction. Therefore $RaR = R$. □

Theorem 2.15. *Let R be a complement left bounded ring.*

(i) *If for every $b \in R, RbR + l(RbR)$ is a complement left ideal of R, then R is reduced biregular.*
(ii) *If for every $a \in R, Ra + l(a)$ is a complement left ideal of R, then R is strongly regular.*

Proof. (i). Let $a \in R$ such that $(RaR)^2 = 0$. Then $RaR \subseteq l(RaR)$. Suppose $l(RaR)$ is not an essential left ideal of R, then there exists a non-zero complement left ideal L of R such that $l(RaR) \oplus L$ is an essential left ideal of R. By hypothesis, L contains a non-zero ideal I of R. Now

$$IRaR \subseteq I \cap RaR \subseteq I \cap l(RaR) = 0, \; I \subseteq l(RaR) \cap L = 0.$$

This contradicts that $I \neq 0$. So $l(RaR)$ is an essential left ideal of R. By hypothesis $l(RaR)$ is a complement left ideal of R. It follows that $l(RaR) = R$. This implies that $RaR = 0$. This yields R is semiprime. Hence for every $c \in R$, we have $RcR \cap l(RcR) = 0$. Since R is complement left bounded and $RcR + l(RcR)$ is a complement left ideal of R, it is easy to see that $RcR \oplus l(RcR) = R$. Hence $RcR = Re$ for some $e \in I(R)$. Since R is semiprime it follows that e is central.

This shows that R is biregular. Hence by Proposition 2.12, we get R is reduced.
(ii). Let $b \in R$ such that $l(b) + Rb \neq R$. If $l(b) + Rb \neq R$ is not essential, then there exists a non-zero complement left ideal K of R such that $(l(b) + Rb) \oplus K$ is an essential left ideal of R. Since R is complement left bounded, it is easy to see that $K = 0$. Hence $l(b) + Rb$ is essential. Since $l(b) + Rb$ is also a complement left ideal of R, it follows that $l(b) + Rb = R$. This shows that R is strongly regular. □

Proposition 2.16. *If every maximal left ideal of R is regular, then R is regular*

Proof. Let $a \in R$. If $Ra = R$, then $a = aba$ for some $b \in R$. If $Ra \neq R$, there exists a maximal left ideal M of R containing Ra. Since M is regular and $a \in M$, there exists some $c \in R$ such that $a = aca$. This shows that R is regular. □

Lemma 2.17. [5, Lemma 3.14] *Let L be a left ideal of R. Then R/L is a flat left R-module if and only if for every $a \in L$, there exists some $b \in L$ such that $a = ab$.*

Notation: We write $N_1(R) = \{a \in R : a^2 = 0\}$.

Proposition 2.18. *Let R be a ring such that $N_1(R)$ is regular. If every simple singular left R-module is flat or YJ-injective, then R is semiprimitive.*

Proof. Let $b \in J(R)$ such that $b^2 = 0$. By hypothesis, there exists some $c \in R$ such that $b = bcb$, that is $(1 - bc)b = 0$. As $b \in J(R)$, $1 - bc$ is invertible, hence $b = 0$. This proves that $J(R)$ is reduced. Hence $l(d) = r(d)$ for every $d \in J(R)$. Let $u \in J(R)$ such that $l(u) + Ru \neq R$. Then there exists a maximal left ideal M of R such that $l(u) + Ru \subseteq M$. If M is not essential then $M = l(e)$ for some $0 \neq e \in I(R)$. Since $u \in M = l(e), ue = 0$. Hence $e \in r(u) = l(u) \subseteq M = l(e)$. Thus $e = e^2 = 0$ a contradiction. Therefore M is essential. Hence by hypothesis R/M is flat or YJ-injective. If R/M is flat, since $u \in M$, by Lemma 2.17, there exists some $v \in M$ such that $u = uv$, that is $1 - v \in r(u) = l(u) \subseteq M$, whence $1 \in M$, a contradiction. If R/M is YJ-injective, it follows that that there exists a positive integer n such that $1 - u^n v \in M$. But $u^n v \in J(R) \subseteq M$, whence $1 \in M$, again a contradiction. Hence $l(u) + Ru = R$ and so $x + yu = 1$ for some $x \in R$, $y \in l(u)$, yielding $yu^2 = u$. As $u \in J(R)$, $1 - yu$ is invertible, hence $u = 0$. This shows that $J(R) = 0$. □

Proposition 2.19. *Let R be a ring such that $N_1(R)$ is regular. Then R is left and right non-singular.*

Proof. If $Z(_RR) \neq 0$, then there exists some $0 \neq a \in Z(_RR)$ such that $a^2 = 0$. By hypothesis, there exists some $x \in R$ such that $a = axa$. Then $xa \in I(R)$. Since $Z(_RR)$ cannot contain a non-zero idempotent, it follows that $xa = 0$, whence $a = axa = 0$. This contradicts that $a \neq 0$. Hence $Z(_RR) = 0$. Similarly we can prove that $Z(R_R) = 0$. □

Lemma 2.20. [5, Remark 3.13] *A reduced left (right) SF-ring is strongly regular.*

Theorem 2.21. *Let R be a left SF-ring such that $r(x)^n$ is an ideal for all $x \in N_1(R)$ and some positive integer n (n depending on x). Then R is strongly regular.*

Proof. Let $0 \neq b \in R$ such that $b^2 = 0$. By hypothesis there exists a positive integer m such that $r(b)^m$ is an ideal. If $Rr(b) \neq R$, then there exists a maximal left ideal M of R such that $Rr(b) \subseteq M$. Since $b \in r(b) \subseteq Rr(b) \subseteq M$ and R is left SF, there exists some $c \in M$ such that $b = bc$, that is $1 - c \in r(b) \subseteq M$, whence $1 \in M$, a contradiction to $M \neq R$. Hence $Rr(b) = R$. This yields $r(b) = r(b)^2$ and so $r(b) = r(b)^m$. Therefore $r(b)$ is an ideal of R. It follows that $R = Rr(b) \subseteq r(b)$ and so $b = 0$. This proves that R is reduced so that by Lemma 2.20, R is strongly regular. □

Corollary 2.22. *Let R be a left SF-ring such that $r(x)$ is nilpotent for every $0 \neq x \in N_1(R)$. Then R is strongly regular.*

Corollary 2.23. [11, Theorem 10] *The following conditions are equivalent for a ring R.*

(1) *R is strongly regular.*
(2) *R is a ZI-ring whose simple left modules are flat.*

References

[1] H.E. Abulkheir and G.F. Birkenmeier, *Right complement bounded semiprime rings*, Acta. Math. Hungar., 70(3) (1996), 227-235.

[2] J. Chen and N. Ding, *On regularity of rings*, Algebra Colloq., 8(3) (2001), 267-274.

[3] X. Guangshi, *On GP-V-rings and characterizations of strongly regular rings*, Northeast. Math. J., 18(4) (2002), 291-297.

[4] Y. Hirano and H. Tominaga, *Regular rings, V-rings and their generalizations*, Hiroshima Math. J., 9 (1979), 137-149.

[5] M. B. Rege, *On von Neumann regular rings and SF-rings*, Math. Japonica, 31(6) (1986), 927-936.

[6] T. Subedi and A.M. Buhphang, *On weakly regular rings and generalizations of V-rings*, Int. Electron. J. Algebra, 10 (2011), 162-173.

[7] R. Yue Chi Ming, *On von Neumann regular rings*, Proc. Edinburgh Math. Soc., 19 (1974), 89-91.

[8] R. Yue Chi Ming, *On quasi injectivity and von Neumann regularity*, Monatsh. Math., 95 (1983), 25-32.

[9] R. Yue Chi Ming, *On biregularity and regularity*, Comm. Algebra, 20 (1992), 749-759.

[10] R. Yue Chi Ming, *On injectivity and p-injectivity, II*, Soochow J. Math., 21 (1995), 401-412.

[11] R. Yue Chi Ming, *On rings close to regular and p-injectivity*, Comment. Math. Univ. Carolin., 47(2) (2006), 203-212.

[12] H. Zhou, *Left SF-rings and regular rings*, Comm. Algebra, 35 (2007), 3842-3850.

Tikaram Subedi
Department of Mathematics
National Institute of Technology Meghalaya
Shillong, India.
email: tsubedi2010@gmail.com

A. M. Buhpang
Department of Mathematics
North Eastern Hill University
Shillong, India.
e-mail: ardeline17@gmail.com