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CHARACTERIZATIONS OF (σ, τ)-GENERALIZED JORDAN DERIVATIONS ON PRIME RINGS

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Abstract. In this paper, we characterize (σ, τ)-generalized Jordan derivations from a ring R into an S-bimodule X, where $\sigma, \tau \colon R \longrightarrow S$ are ring homomorp-hisms. Our result covers a known result due to Nakajima [Turkish J. Math., 30 (2006), 403-411].

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1. Introduction

Let R be a ring and X be an R-bimodule. An additive map $\delta : R \longrightarrow X$ is called a *derivation* if it satisfies

$$\delta(ab) = \delta(a)b + a\delta(b), \quad a, b \in R. \tag{1}$$

If the equality (1) only hold in the case where $b = a$, then δ is called a *Jordan derivation*. We denote by $[a, b]$, the commutator $ab - ba$. Each mapping of the form $a \longmapsto [a, x]$, where $x \in X$, will be called an inner derivation. Clearly, every derivation is Jordan derivation, however, there exists Jordan derivations which are not derivations, see [3,7].

Recall that a ring R is called *prime* if $aRb = 0$ implies that $a = 0$ or $b = 0$, and it is called *semiprime* if $aRa = 0$ implies $a = 0$. A classical result of Herstein [6] states that every Jordan derivation from a 2-torsion free prime ring into itself is a derivation and it was extended to 2-torsion free semiprime rings by Brešar [2]. Johnson [7] proved that every continuous Jordan derivation δ from a C^*-algebra A into any Banach A-bimodule X is a derivation. Of course, the continuity of δ can be removed, see [9]. Zhang [11] proved that every Jordan derivation on nest algebras is an inner derivation. In [5], the authors proved that each Jordan derivation on a triangular ring is a derivation.

Let R and S be rings, X be an S-bimodule and let $\sigma, \tau : R \longrightarrow S$ be additive maps. A biadditive map $\mu : R \times R \longrightarrow X$ is said to be a (σ, τ)-*Hochschild* 2-*cocycle* if

$$\sigma(a)\mu(b, c) - \mu(ab, c) + \mu(a, bc) - \mu(a, b)\tau(c) = 0, \qquad a, b, c \in R.$$

A (σ, τ)-Hochschild 2-cocycle map μ is called *symmetric* if $\mu(a, b) = \mu(b, a)$ for all $a, b \in R$.

An additive map $\delta : R \longrightarrow X$ is said to be a (σ, τ)-*generalized derivation* if there exists a (σ, τ)-Hochschild 2-cocycle μ such that for all $a, b \in R$,

$$\delta(ab) = \delta(a)\tau(b) + \sigma(a)\delta(b) + \mu(a, b),$$

and it is called a (σ, τ)-*generalized Jordan derivation* if

$$\delta(a^2) = \delta(a)\tau(a) + \sigma(a)\delta(a) + \mu(a, a), \qquad a \in R.$$

The concept of (σ, τ)-generalized derivation associated with a (σ, τ)-Hochschild 2-cocycle was introduced by Zhou [12], as an extension of generalized derivation associated with a Hochschild 2-cocycle μ. Indeed, if $R = S$ and $\sigma = \tau = \text{id}$, the identity map on R, then (σ, τ)-generalized derivation is simply called a generalized derivation which was introduced by Nakajima [8]. Moreover, if $\mu = 0$, then they are the usual derivations and Jordan derivations, respectively.

Next we show that the class of (σ, τ)-generalized derivations is large. Indeed, it contains τ-multipliers, (σ, τ)-derivations and all another type of generalized derivations.

We mention that in the next example $\sigma, \tau : R \longrightarrow S$ are ring homomorphisms.

Example 1.1. (i) Suppose that δ satisfies $\delta(ab) = \delta(a)\tau(b) + \sigma(a)d(b)$, where $d : R \longrightarrow X$ is a (σ, τ)-derivation. Then the map $\mu_1 : R \times R \longrightarrow X$ via $\mu_1(a, b) = \sigma(a)(d - \delta)(b)$ is biadditive and it is (σ, τ)-Hochschild 2-cocycle. Moreover, for all $a, b \in R$,

$$\delta(ab) = \delta(a)\tau(b) + \sigma(a)\delta(b) + \mu_1(a, b).$$

Thus, δ is a (σ, τ)-generalized derivation associated with μ_1.

(ii) Suppose that $\delta : R \longrightarrow X$ is a left τ-multiplier, that is, $\delta(ab) = \delta(a)\tau(b)$. Then by the equality $\delta(ab) = \delta(a)\tau(b) + \sigma(a)\delta(b) + \sigma(a)(-\delta)(b)$, we have a (σ, τ)-Hochschild 2-cocycle biadditive map $\mu_2 : R \times R \longrightarrow X$ defined by $\mu_2(a, b) = \sigma(a)(-\delta)(b)$. Thus, a left τ-multiplier is also a (σ, τ)-generalized derivation.

(iii) Let δ satisfy the relation $\delta(ab)=\delta(a)\sigma(b)+\tau(a)\delta(b)$ for all $a,b\in R$. Then the map $\mu_3 : R\times R\longrightarrow X$ defined by

$$\mu_3(a,b)=\delta(a)\big(\sigma(b)-\tau(b)\big)+\big(\tau(a)-\sigma(a)\big)\delta(b),$$

is (σ,τ)-Hochschild 2-cocycle and

$$\delta(ab)=\delta(a)\tau(b)+\sigma(a)\delta(b)+\mu_3(a,b).$$

Hence a (τ,σ)-derivation is also a (σ,τ)-generalized derivation.

The following theorem was proved by Nakajima in [8].

Theorem 1.2. *Suppose that R is a 2-torsion free ring and $\delta : R\longrightarrow R$ is a generalized Jordan derivation associate with Hochschild 2-cocycle μ. If R satisfies one of the following conditions, then δ is a generalized derivation.*

(i) *R is a non-commutative prime ring,*
(ii) *There exist $a,b\in R$ such that $[a,b]$ is a non-zero divisor,*
(iii) *R is commutative and μ is symmetric.*

The aim of this paper is to generalize Theorem 1.2 for (σ,τ)-generalized Jordan derivations from a ring R into an S-bimodule X. Note that our approach is quite different from that in [8].

Throughout this paper, R and S are rings, X is an S-bimodule and $\sigma,\tau : R\longrightarrow S$ are ring homomorphisms.

2. Main results

In this section, we characterize (σ,τ)-generalized Jordan derivations $\delta : R\longrightarrow X$ and prove under special hypothesis that such maps necessary are (σ,τ)-generalized derivations.

For all $a,b\in R$, we introduce the notation

$$D(a,b)=\delta(ab)-\delta(a)\tau(b)-\sigma(a)\delta(b)-\mu(a,b).$$

Using the same approach as in the proof of [8, Lemmas 2 and 4], we have

Lemma 2.1. *Let R and S be rings and X be a 2-torsion free S-bimodule. If $\delta : R\longrightarrow X$ is a (σ,τ)-generalized Jordan derivation, then*

(i) $\delta(ab+ba)=\delta(a)\tau(b)+\sigma(a)\delta(b)+\mu(a,b)+\delta(b)\tau(a)+\sigma(b)\delta(a)+\mu(b,a),$
(ii) $\delta(aba)=\delta(a)\tau(ba)+\sigma(a)\delta(b)\tau(a)+\sigma(ab)\delta(a)+\sigma(a)\mu(b,a)+\mu(a,ba),$
(iii) $\delta(abc+cba)=\delta(a)\tau(bc)+\sigma(a)\delta(b)\tau(c)+\sigma(ab)\delta(c)+\sigma(a)\mu(b,c)+\mu(a,bc)$

$$+\delta(c)\tau(ba)+\sigma(c)\delta(b)\tau(a)+\sigma(cb)\delta(a)+\sigma(c)\mu(b,a)+\mu(c,ba),$$

(iv) *$D(a,b)\tau(c)[\tau(a),\tau(b)]+[\sigma(a),\sigma(b)]\sigma(c)D(a,b)=0$,*

(v) *$D(a,b)[\tau(a),\tau(b)]=0$, and $[\sigma(a),\sigma(b)]D(a,b)=0$.*

For the proof of the main theorem, we need the following lemma.

Lemma 2.2. [4, Lemma 4] *Let G and H be additive groups and let R be a 2-torsion free ring. Let $f: G\times G\longrightarrow H$ and $h: G\times G\longrightarrow R$ be biadditive maps. Suppose that for each pair $a,b\in G$ either $f(a,b)=0$ or $h(a,b)^2=0$. Then either $f(a,b)=0$ for all $a,b\in G$, or $h(a,b)^2=0$ for all $a,b\in G$.*

Remark 2.3. [4, Remark 5] It is worth noting that if a ring S and a nonzero S-bimodule X are such that $xSa=0$ with $x\in X$, $a\in S$ implies that $x=0$ or $a=0$, then S is prime. Indeed, suppose that $aSb=0$ for some $a,b\in S$. Then for any nonzero $x\in X$ we have $(xSa)Sb=0$, and hence it follows that $a=0$ or $b=0$.

Moreover, if X is 2-torsion free, then S is 2-torsion free. To see this let $2a=0$ for some $a\in S$. Then $2xSa=0$ for all $x\in X$ and so $a=0$.

Our first main theorem is stated as follows and serves as a generalization of Theorem 1.2(i).

Theorem 2.4. *Let R be any ring, S be a noncommutative ring and X be a 2-torsion free S-bimodule. Suppose that either*

(i) *τ is onto and $xSa=0$ with $x\in X$, $a\in S$ implies that $x=0$ or $a=0$, or*

(ii) *σ is onto and $aSx=0$ with $x\in X$, $a\in S$ implies that $x=0$ or $a=0$.*

In this case each (σ,τ)-generalized Jordan derivation δ from R into X is a (σ,τ)-generalized derivation.

Proof. We only prove the case where τ is onto and $xSa=0$ with $x\in X$, $a\in S$ implies that $x=0$ or $a=0$. The case (ii) can be discussed analogously.

Multiply the relation (iv) in Lemma 2.1 from the right by $[\tau(a),\tau(b)]$. According to (v) in Lemma 2.1, for all $a,b\in R$, we obtain

$$D(a,b)\tau(c)[\tau(a),\tau(b)]^2=0.$$

Since τ is onto, our assumption implies that for each pair $a,b\in R$ either $D(a,b)=0$ or $[\tau(a),\tau(b)]^2=0$. It is by Remark 2.3 that S is 2-torsion free. Applying Lemma 2.2 for the mapping $f(a,b)=D(a,b)$ and $h(a,b)=[\tau(a),\tau(b)]$, we get either $D(a,b)=0$ for all $a,b\in R$ or $[\tau(a),\tau(b)]^2=0$ for all $a,b\in R$.

Suppose that $D(a,b)\neq 0$ for some $a,b\in R$. Then $[\tau(a),\tau(b)]^2=0$ for every $a,b\in R$. Since τ is onto, we conclude that $[x,y]^2=0$ for all $x,y\in S$. By Remark 2.3, S is a prime ring. Then it follows from [10, Lemma] that S is commutative, which is contradiction. Consequently, $D(a,b) = 0$ for all $a, b \in R$ and hence $\delta\colon R \longrightarrow X$ is a (σ,τ)-generalized derivation. □

Take $R = S = X$ in Theorem 2.4, we get the following result.

Corollary 2.5. *Suppose that R is a 2-torsion free noncommutative prime ring. If τ is surjective (or σ is surjective), then every (σ,τ)-generalized Jordan derivation δ on R is a (σ,τ)-generalized derivation.*

If $\sigma = \tau = \mathrm{id}$ in Corollary 2.5, then we obtain the next corollary.

Corollary 2.6. [8, Theorem 6] *If R is a 2-torsion free noncommutative prime ring, then every generalized Jordan derivation $\delta : R \longrightarrow R$ is a generalized derivation.*

The condition that $xSa = 0$ with $x \in X$, $a \in S$ implies that $x = 0$ or $a = 0$, in Theorem 2.4 is essential. The following example illustrates this fact.

Example 2.7. Let

$$R = \left\{ \begin{bmatrix} z_1 & z_2 \\ 0 & z_3 \end{bmatrix} : \ z_1, z_2, z_3 \in \mathbb{C} \right\}.$$

We make $X = \mathbb{C}$ an R-bimodule by defining

$$a\lambda = z_3\lambda, \quad \lambda a = \lambda z_1, \quad \lambda \in \mathbb{C}, \ a \in R.$$

Define $\delta : R \longrightarrow X$ via $\delta(a) = z_2$ for all $a \in R$. Then

$$\delta(a^2) = \delta(a)a + a\delta(a)$$

for all $a \in R$. Therefore, δ is a generalized Jordan derivation associated with Hochschild 2-cocycle $\mu = 0$. However, δ is not a generalized derivation.

Note that the condition $\lambda Ra = 0$ with $\lambda \in X = \mathbb{C}$, $a \in R$ does not imply that $\lambda = 0$ or $a = 0$.

It is proved in [1, Theorem 1] that if R is a 2-torsion free semiprime ring, τ is surjective and $\tau(Z(R)) = Z(R)$, where $Z(R)$ is the center of R, then each left Jordan τ-multiplier $\delta : R \longrightarrow R$ is a left τ-multiplier. For another characterization of τ-multipliers, see [13,14] and the references therein.

Next we consider this result in two different cases. In the first case we assume that R is commutative and outline a new simple proof for it as follows.

Theorem 2.8. *Let R be a 2-torsion free commutative semiprime ring. If τ is surjective, then each left Jordan τ-multiplier $\delta : R \longrightarrow R$ is a left τ-multiplier.*

Proof. By our assumption,

$$\delta(a^2) = \delta(a)\tau(a), \qquad a \in R.$$

Replacing a by $a+b$, we get

$$2\delta(ab) = \delta(a)\tau(b) + \delta(b)\tau(a), \qquad a, b \in R. \tag{2}$$

Interchanging b by bc in (2), we obtain

$$2\delta(abc) = \delta(a)\tau(bc) + \delta(bc)\tau(a). \tag{3}$$

Plugging (2) into (3) to get

$$4\delta(abc) = 2\delta(a)\tau(b)\tau(c) + \big(\delta(b)\tau(c) + \delta(c)\tau(b)\big)\tau(a). \tag{4}$$

Similarly,

$$4\delta(bac) = 2\delta(b)\tau(a)\tau(c) + \big(\delta(a)\tau(c) + \delta(c)\tau(a)\big)\tau(b). \tag{5}$$

Comparing (4) and (5) and using the fact that $\tau(a)\tau(b) = \tau(b)\tau(a)$ for all $a, b \in R$, we arrive at

$$\big(\delta(a)\tau(b) - \delta(b)\tau(a)\big)\tau(c) = 0, \qquad a, b, c \in R. \tag{6}$$

Multiplying the relation (6) from the right by $\big(\delta(a)\tau(b) - \delta(b)\tau(a)\big)$, we get

$$\big(\delta(a)\tau(b) - \delta(b)\tau(a)\big)\tau(c)\big(\delta(a)\tau(b) - \delta(b)\tau(a)\big) = 0.$$

Since R is semiprime and τ is surjective, we conclude that $\delta(a)\tau(b) - \delta(b)\tau(a) = 0$ for all $a, b \in R$. Thus, it follows from (2) that $\delta(ab) = \delta(a)\tau(b)$ for all $a, b \in R$ and hence δ is a left τ-multiplier. □

In the second case we consider the noncommutative situation and relaxing the condition $\tau(Z(R)) = Z(R)$, but we assume the stronger condition that R is prime.

Corollary 2.9. *Suppose that R is a 2-torsion free noncommutative prime ring. If τ is surjective, then each left Jordan τ-multiplier $\delta : R \longrightarrow R$ is a left τ-multiplier.*

Proof. Take $\sigma = \mu = 0$ in Corollary 2.5. □

Let R be a commutative ring, $\sigma = \tau$ and μ is a symmetric (σ, τ)-Hochschild 2-cocycle map. Then by Lemma 2.1(i), every (σ, τ)-generalized Jordan derivation $\delta : R \longrightarrow R$ is a (σ, τ)-generalized derivation. The following result improve this conclusion.

Recall that an S-bimodule X is said to be *symmetric* if $ax = xa$ for all $a \in S$ and $x \in X$.

Theorem 2.10. *Let R be a commutative ring and S be any ring. Let X be a 2-torsion free symmetric S-bimodule with the property that $xa = 0$ with $x \in X$, $a \in S$ implies that $x = 0$ or $a = 0$. If μ is symmetric, then each (σ,τ)-generalized Jordan derivation $\delta : R \longrightarrow X$ is a (σ,τ)-generalized derivation.*

Proof. Let $\delta : R \longrightarrow X$ be a (σ,τ)-generalized Jordan derivation. Then

$$\delta(a^2) = \delta(a)\tau(a) + \sigma(a)\delta(a) + \mu(a,a), \quad a \in R. \tag{7}$$

Replacing a by a^2 in Lemma 2.1(i), we get

$$2\delta(a^2b) = \delta(a^2)\big(\tau(b) + \sigma(b)\big) + \delta(b)\big(\sigma(a^2) + \tau(a^2)\big) + \mu(a^2,b) + \mu(b,a^2), \tag{8}$$

for all $a, b \in R$. By (7) and (8),

$$\begin{aligned}2\delta(a^2b) =&\delta(a)\tau(a)\tau(b) + \sigma(a)\delta(a)\tau(b) + \mu(a,a)\tau(b)\\ &+\delta(a)\tau(a)\sigma(b) + \sigma(a)\delta(a)\sigma(b) + \mu(a,a)\sigma(b)\\ &+\delta(b)\sigma(a)\sigma(a) + \delta(b)\tau(a)\tau(a) + \mu(a^2,b) + \mu(b,a^2).\end{aligned}$$

On the other hand, according to (ii) in Lemma 2.1, we have

$$\begin{aligned}2\delta(a^2b) =&2\delta(a)\tau(b)\tau(a) + 2\sigma(a)\delta(a)\tau(b) + 2\sigma(a)\sigma(b)\delta(a)\\ &+2\sigma(a)\mu(b,a) + 2\mu(a,ba).\end{aligned}$$

Comparing the above two expressions, we obtain

$$\begin{aligned}&\big(\delta(a)\tau(b) + \sigma(a)\delta(b) - \delta(a)\sigma(b) - \tau(a)\delta(b)\big)\big(\sigma(a) - \tau(a)\big)\\ &+\big(\mu(a,a)\tau(b) + \mu(a^2,b) - \mu(a,ba)\big) - \mu(a,ba)\\ &+\big(\sigma(b)\mu(a,a) + \mu(b,a^2) - \sigma(a)\mu(b,a)\big) - \sigma(a)\mu(b,a) = 0.\end{aligned} \tag{9}$$

Since μ is a (σ,τ)-Hochschild 2-cocycle map, we have the following relation:

(i) $\sigma(a)\mu(b,a) + \mu(a,ba) = \mu(ab,a) + \mu(a,b)\tau(a)$,
(ii) $\mu(a,a)\tau(b) + \mu(a^2,b) - \mu(a,ab) = \sigma(a)\mu(a,b)$,
(iii) $\sigma(b)\mu(a,a) + \mu(b,a^2) - \mu(b,a)\tau(a) = \mu(ba,a)$.

Since R is commutative and μ is symmetric, by (i) we get

$$\sigma(a)\mu(b,a) = \mu(a,b)\tau(a), \quad a, b \in R,$$

and hence (iii) implies that

$$\sigma(b)\mu(a,a) + \mu(b,a^2) - \sigma(a)\mu(b,a) = \mu(ba,a), \quad a, b \in R. \tag{10}$$

Plugging the relation (ii) and (10) into (9), we get

$$\big(\delta(a)\tau(b) + \sigma(a)\delta(b) - \delta(a)\sigma(b) - \tau(a)\delta(b)\big)\big(\sigma(a) - \tau(a)\big) = 0. \tag{11}$$

By our assumption, it follows from (11) that for each $a \in R$ either $\sigma(a) = \tau(a)$ or for all $b \in R$,

$$\delta(a)\tau(b) + \sigma(a)\delta(b) = \delta(a)\sigma(b) + \tau(a)\delta(b).$$

In other words, R is the union of its subsets $A = \{a \in R : \ \sigma(a) = \tau(a)\}$ and

$$B = \{a \in R : \ \delta(a)\tau(b) + \sigma(a)\delta(b) = \delta(a)\sigma(b) + \tau(a)\delta(b), \ \text{for all } b \in R\}.$$

Clearly, each of A and B are additive subgroups of R. But a group cannot be the union of two proper subgroups, therefore $A = R$ or $B = R$.

If $A = R$, then $\sigma = \tau$ and hence from (i) in Lemma 2.1, it follows that $\delta : R \longrightarrow X$ is a (σ, τ)-generalized derivation.

If $B = R$, then for all $a, b \in R$, we have

$$\delta(a)\tau(b) + \sigma(a)\delta(b) = \delta(a)\sigma(b) + \tau(a)\delta(b).$$

Thus, by using (i) in Lemma 2.1, we see that δ is a (σ, τ)-generalized derivation. □

Corollary 2.11. *Let R be a commutative prime ring (i.e., a commutative integral domain) and $\delta : R \longrightarrow R$ be a (σ, τ)-generalized Jordan derivation. If μ is symmetric, then δ is a (σ, τ)-generalized derivation.*

Proof. Take $R = S = X$ in Theorem 2.10. □

The next example shows that selecting an appropriate (σ, τ)-Hochschild 2-cocycle μ plays a crucial role. Moreover, it shows that the primeness of R can be omitted from Corollary 2.11 whether $\sigma = \tau$.

Example 2.12. Let

$$R = \left\{ \begin{bmatrix} z_1 & z_2 \\ 0 & z_1 \end{bmatrix} : \ z_1, z_2 \in \mathbb{C} \right\}.$$

Then R is a commutative ring. Suppose that $\delta : R \longrightarrow R$ is an additive map defined by $\delta(x) = xm + mx$, where

$$m = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

Let $\sigma, \tau : R \longrightarrow R$ be additive maps with

$$\sigma(a) = \tau(a) = \begin{bmatrix} z_1 & 0 \\ 0 & z_1 \end{bmatrix}, \qquad a \in R.$$

Define $\mu_1, \mu_2 : R \times R \longrightarrow R$ via

$$\mu_1(a, b) = -\sigma(a)\delta(e_A)\tau(b), \quad \mu_2\left(\begin{bmatrix} z_1 & z_2 \\ 0 & z_1 \end{bmatrix}, \begin{bmatrix} w_1 & w_2 \\ 0 & w_1 \end{bmatrix} \right) = \begin{bmatrix} 0 & -z_1 w_1 \\ 0 & 0 \end{bmatrix}.$$

Then both μ_1 and μ_2 are (σ, τ)-Hochschild 2-cocycle and they are symmetric. Since

$$\delta(a^2) = \delta(a)\tau(a) + \sigma(a)\delta(a) + \mu_1(a, a),$$

for all $a \in R$ and $\sigma = \tau$, δ is a (σ, τ)-generalized derivation associated with μ_1, but δ is not a (σ, τ)-generalized Jordan derivation associated with μ_2.

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