International Electronic Journal of Algebra
Volume 6 (2009) 38-45

DIFFERENTIAL POLYNOMIALS OVER BAER RINGS

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Received: 26 May 2008; Revised: 19 May 2009
Communicated by Syed M. Tariq Rizvi

Abstract. Let R be a ring with unity and δ a derivation on R. In this paper we extend a result of Armendariz on the Baer condition in a polynomial ring to a Baer condition in a nearring of differential polynomial. The nearring of differential has substitution for its "multiplication" operation.

Mathematics Subject Classification (2000): 16Y30, 16S36
Keywords: annihilator conditions, nearrings, derivation polynomial rings, Baer rings, Armendariz rings

1. Introduction

Throughout this paper all rings are associative and all nearrings are left nearrings. We use R and N to denote a ring and a nearring respectively. The study of Rickart rings has its roots in both functional analysis and homological algebra. In [18], Rickart studied C*-algebra with the property that every right annihilator of any element is generated by a projection (an idempotent p is called a projection if $p = p^*$, where $*$ is an involution on the algebra). This condition is modified by Kaplansky [16] through introducing *Baer* rings (a ring R is called Baer if the right annihilator of every nonempty subset of R is generated, as a right ideal, by an idempotent of R) to abstract various properties of AW*-algebra and von Neumann algebra. See also Berberian [2] for more details.

A ring satisfying a generalization of *Rickart's condition* (i.e., every right annihilator of any element in R, as a right ideal, by an idempotent) has a homological characterization as a right *PP ring*, i.e., every principal right ideal is projective. Left PP rings are defined similarly. In [9] Clark defines a ring to be *quasi-Baer* if the left annihilator of every ideal is generated, as a left ideal, by an idempotent. Then he used the quasi-Baer concept to characterize when a finite-dimensional algebra with unity over an algebraically closed field is isomorphic to a twisted matrix units semigroup algebra. Every prime ring is a quasi-Baer ring. It is natural to ask if some of these properties can extended from a ring R to the polynomial ring

This research is supported by the Shahrood University of Technology at Iran.

$(R[x], +, .)$ and vice versa. Armendariz [1] and Birkenmeier et al. [5] obtained the following results:

Theorem B [1]. Let R be a *reduced ring* (i.e. R has no nonzero nilpotent element). Then R is a Baer (resp. PP) ring if and only if $(R[x], +, .)$ is a Baer (resp. PP) ring.

Theorem [5]. R is quasi-Baer if and only if $R[x]$ is quasi-Baer.

Armendariz provided an example to show that the reduced condition is not superfluous. A generalization of Armendariz's result for several types of polynomial extensions over Baer and quasi-Baer rings, are obtained by various authors [5,6,7, 12,14,15]. In [15] Hong et al. studied Ore extension of Baer and quasi-Baer rings.

Three commonly used operations for polynomials are addition " $+$ ", multiplication "." and substitution " $\circ$ " [8,10,11,17], respectively. Observe that $(R[x], +, .)$ is a ring and $(R[x], +, \circ)$ is a left nearring where the substitution indicates substitution of $f(x)$ into $g(x)$, explicitly $f(x) \circ g(x) = ((x)f)g$ for each $f(x), g(x) \in R[x]$. It is natural to investigate the nearring of polynomials $R[x]$ and the zero-symmetric nearring of polynomials $R_0[x]$. Birkenmeier and Huang in [3,4], have defined the *Baer-type annihilator conditions* in the class of nearrings as follows (for a nonempty $S \subseteq N$, let $r_N(S) = \{a \in N \mid Sa = 0\}$ and $\ell_N(S) = \{a \in N \mid aS = 0\}$):

(1) $N \in \mathcal{B}_{r1}$ if $r_N(S) = eN$ for some idempotent $e \in N$;

(2) $N \in \mathcal{B}_{r2}$ if $r_N(S) = r_N(e)$ for some idempotent $e \in N$;

(3) $N \in \mathcal{B}_{\ell 1}$ if $\ell_N(S) = Ne$ for some idempotent $e \in N$;

(4) $N \in \mathcal{B}_{\ell 2}$ if $\ell_N(S) = \ell_N(e)$ for some idempotent $e \in N$.

When S is a singleton, the *Rickart-type annihilator conditions* on nearrings are also defined similarly except replacing $\mathcal{B}$ by $\mathcal{R}$. If the subset S considered in the above definition is replaced with an ideal, we obtain the *quasi-Baer annihilator conditions* in the class of nearrings, denoted by "$q\mathcal{B}$" in the above notations. In particular they studied Baer-type conditions on the nearring of polynomials $R[x]$ (with the operations of addition and substitution) and formal power series by obtaining the following results: Let R be a reduced ring. (1) If R is Baer, then $R_0[x]$ (resp. $R_0[[x]]$) satisfies all the Baer-type annihilator conditions. (2) If $R_0[x]$ (resp. $R_0[[x]]$) satisfies any one of the Baer-type annihilator conditions, then R is Baer.

Let δ be a derivation of R, that is, δ is an additive map such that $\delta(ab) = \delta(a)b + a\delta(b)$, for all $a, b \in R$. Since $R[x; \delta]$ is an abelian nearring under addition and substitution, it is natural to investigate the nearring of differential polynomials $(R[x; \delta], +, \circ)$ when R is Baer. We use $R[x; \delta]$ to denote the left nearring of differential polynomials $(R[x; \delta], +, \circ)$ with coefficients from R and $R_0[x; \delta] =$

$\{f \in R[x;\delta] \mid f \text{ has zero constant term}\}$ the 0-symmetric subnearring of $R[x;\delta]$. Let $(x)f = a_0 + a_1x$ and $(x)g = b_0 + b_1x + b_2x^2 \in R[x;\delta]$. Through a simple calculation, we have $(x)f \circ (x)g = ((x)f)g = b_0 + b_1((x)f) + b_2((x)f)^2 = (b_0 + b_1a_0 + b_2a_0^2 + b_2a_1\delta(a_0)) + (b_1a_1 + b_2a_0a_1 + b_2a_1a_0 + b_2a_1\delta(a_1))x + b_2a_1^2x^2$.

2. Main Results

A nearring N is said to have the *insertion of factors property* (or simply IFP) if for all $a, b, n \in N$, $ab = 0$ implies $anb = 0$. Clearly each reduced nearring has the IFP.

Lemma 2.1. *Let R be a reduced ring and $a, b \in R$. Then we have the following:*

(1) *If $ab = 0$, then $a\delta^m(b) = 0 = \delta^m(a)b$ for any positive integers m.*
(2) *If $e^2 = e \in R$, then $\delta(e) = 0$.*

Proof. (1) It is enough to show that $a\delta(b) = \delta(a)b = 0$. If $ab = 0$, then $\delta(ab) = \delta(a)b + a\delta(b) = 0$. Hence $a\delta(a)b + a^2\delta(b) = 0$, and that $a\delta(b) = 0$, since R is reduced and $ab = 0$.

(2) If $e^2 = e$, then $\delta(e) = \delta(e)e + e\delta(e)$. Since R is reduced, so e belong to the center of R. Hence $2e\delta(e) = e\delta(e)$, and that $e\delta(e) = 0$. Thus $\delta(e) = 0$. □

Lemma 2.2. *Let δ be a derivation of a ring R and $R[x;\delta]$ the nearring of differential polynomials over R. Let R be a reduced ring. Then:*

(1) *If $(x)E \in R[x;\delta]$ is an idempotent, then $(x)E = e_1x + e_0$, where e_1 is an idempotent in R with $e_1e_0 = 0$.*
(2) *$R[x;\delta]$ is reduced.*

Proof. (1) Let $(x)E = e_0 + \cdots + e_nx^n$ be an idempotent of $R[x;\delta]$. Since $(x)E \circ (x)E = (x)E$, we have $e_n^{n+1} = 0$, if $n \geq 2$. Thus $e_n = 0$, since R is reduced. Therefore $(x)E = e_0 + e_1x$. Clearly e_1 is an idempotent of R and $e_1e_0 = 0$.

(2) Let $(x)f = a_0 + a_1x + \cdots + a_nx^n \in R[x;\delta]$ such that $(x)f \circ (x)f = 0$. Then $a_n^{n+1} = 0$. Hence $a_n = 0$, since R is reduced. By using induction on n, we have $a_i = 0$ for each $0 \leq i \leq n$. Therefore $(x)f = 0$ and $R[x;\delta]$ is reduced. □

The following example ([3], Example 3.5), shows that there exists a finite reduced commutative Baer ring R such that $R[x] \notin \mathcal{B}_{r2}$.

Example 2.3. *Let $R = Z_6$ and $S = \{2x + 2, 2x + 5\}$. From Lemma 2.2, all idempotents in $Z_6[x]$ are $\{0, 1, 2, 3, 4, 5, x, 3x, 3x + 2, 3x + 4, 4x, 4x + 3\}$. Note that $x - c \in r(c)$ and $x - c \notin r(S)$ for all constant idempotents $c \in Z_6[x]$. Also the possible idempotents $(x)E \in Z_6[x]$ such that $r(S) = r((x)E)$ are either $4x$ or $4x+3$.*

Observe that $3x \in r(4x)$ *but* $3x \notin r(S)$*, and also* $3x^3 + 3 \in r(4x + 3)$ *but* $3x^3 + 3 \notin r(S)$*. Therefore, there is no idempotent* $(x)E \in Z_6[x]$ *such that* $r(S) = r((x)E)$*. Consequently,* $Z_6[x] \notin \mathcal{B}_{r2}$*.*

If $(x)f = \sum_{i=0}^{n} a_i x^i \in R[x;\delta]$, let $S_f^* = \{a_1, a_2, \cdots, a_n\}$.

Proposition 2.4. *Let* R *be a reduced ring. Then:*

(1) $R \in \mathcal{B}_{r1}$ *if and only if* $R_0[x;\delta] \in \mathcal{B}_{\ell 1}$*.*

(2) $R \in \mathcal{B}_{r2}$ *if and only if* $R_0[x;\delta] \in \mathcal{B}_{\ell 2}$*.*

Proof. (1) Assume $R \in \mathcal{B}_{r1}$. Let S be a nonempty subset of $R_0[x;\delta]$. Then $T = \cup_{f\in S} S_f^*$ is a nonempty subset of R. Hence $r_R(T) = eR$ for some idempotent $e \in R$, since $R \in \mathcal{B}_{r1}$. We show that $\ell(S) = R_0[x;\delta] \circ (ex) = e \cdot R_0[x;\delta]$. Let $(x)f = \sum_{i=1}^{m} a_i x^i \in S$. Since $\delta(e) = 0$, we have $(ex) \circ (x)f = \sum_{i=1}^{m} a_i (ex)^i = \sum_{i=1}^{m} a_i e x^i = 0$. Thus $ex \in \ell(S)$ and hence $e \cdot R_0[x;\delta] \subseteq \ell(S)$. Now, let $(x)h = \sum_{k=1}^{n} c_k x^k \in \ell(S)$ and $(x)f = a_1 x \cdots + a_m x^m \in S$. Then $\sum_{i=1}^{m} a_i ((x)h)^i = 0$ and that $a_m (c_n)^m = 0$. Hence $a_m c_n = c_n a_m = 0$, since R is reduced. Thus $\sum_{i=0}^{m-1} a_i c_n ((x)h)^i = 0$ and that $(x)h \circ (c_n a_1 x + \cdots + c_n a_{m-1} x^{m-1}) = 0$. By using induction on $m + n$, we have $a_i c_j = 0$ for $1 \leq i \leq m-1$ and $1 \leq j \leq n$. Therefore $c_k = e c_k$ for all $1 \leq k \leq n$. Hence $(x)h = e \sum_{k=1}^{n} c_k x^k \in eR_0[x;\delta]$ and so $\ell(S) = R_0[x;\delta] \circ (ex)$. Thus $R_0[x;\delta] \in \mathcal{B}_{\ell 1}$.

Now, assume $R_0[x;\delta] \in \mathcal{B}_{\ell 1}$. Let S be a nonempty subset of R, and define $S_x = \{sx | s \in S\}$ a subset of $R_0[x;\delta]$. Then $\ell(S_x) = R_0[x;\delta] \circ (ex)$ for some idempotent $e \in R$, by Lemma 2.2. For each $sx \in S_x$, $0 = (ex) \circ (sx) = sex$. Therefore $e \in r_R(S)$. Now, let $a \in r_R(S)$. Then $(ax) \circ (sx) = sax = 0$ for each $sx \in S_x$. Thus $ax \in \ell(S_x) = R_0[x;\delta] \circ (ex) = e \cdot R_0[x;\delta]$. Hence $a = ea \in eR$. Thus $r_R(S) = eR$. Therefore $R \in \mathcal{B}_{r1}$.

(2) Assume $R \in \mathcal{B}_{r2}$. Let S be a nonempty subset of $R_0[x;\delta]$. By a similar construction to that used in (1), we have $r_R(T) = r_R(e)$ for some idempotent $e \in R$. We claim $\ell(S) = \ell(ex)$. Let $(x)g = \sum_{j=1}^{n} b_j x^j \in \ell(ex)$. Then $0 = (x)g \circ ex = e \cdot (x)g$. Hence $eb_j = 0$ for all $1 \leq j \leq n$. Consequently, $b_j \in r_R(e) = r_R(T)$, for all $1 \leq j \leq n$. Let $(x)f = \sum_{i=1}^{m} a_i x^i \in S$. Then $(x)g \circ (x)f = \sum_{i=1}^{m} a_i (\sum_{j=1}^{n} b_j x^j)^i = 0$. Therefore $\ell(ex) \subseteq \ell(S)$. Now, let $(x)g = \sum_{j=1}^{n} b_j x^j \in \ell(S)$. Then by a similar way as used in (1), $b_j \in r_R(T) = r_R(e)$ for all $1 \leq j \leq n$. Thus $(x)g \circ (ex) = e \cdot (x)g = 0$. Therefore $\ell(S) = \ell(ex)$ and so $R_0[x;\delta] \in \mathcal{B}_{\ell 2}$.

Assume $R_0[x;\delta] \in \mathcal{B}_{\ell 2}$. Let S be a nonempty subset of R and let $S_x = \{sx | s \in S\}$. Then $\ell(S_x) = \ell((x)E)$ for some idempotent $(x)E = ex \in R_0[x;\delta]$, by Lemma 2.2. We show that $r_R(S) = r_R(e)$. Let $a \in r_R(S)$. Then $ax \circ sx = sax = 0$ for all

$sx \in S_x$. Hence $ax \in \ell(S_x) = \ell((x)E)$. Thus $ax \circ ex = eax = 0$ and that $a \in r_R(e)$. Therefore $r_R(S) \subseteq r_R(e)$. Now, let $b \in r_R(e)$. Then $bx \circ ex = ebx = 0$ and that $bx \in \ell(S_x)$. Thus $bx \circ sx = sbx = 0$ for all $s \in S$. Hence $b \in r_R(S)$. Therefore $R \in \mathcal{B}_{r2}$. □

Corollary 2.5. *Let R be a reduced ring. Then the following are equivalent:*

(1) *R is Baer;*
(2) *$(R[x;\delta], +, \cdot)$ is Baer;*
(3) *$(R_0[x;\delta], +, \circ) \in \mathcal{B}_{\ell 1} \cup \mathcal{B}_{\ell 2}$.*

Proof. (1) ⇔ (2) follows from [9], and (1) ⇔ (3) follows from Proposition 2.4. □

Example 2.6. *One can show that the following nearrings satisfy all the Baer-type annihilator conditions discussed in this paper when R is reduced Baer ring: (i) $\{ax \mid a \in R\}$; (ii) $\{(x)f = \sum_{i=1}^{n} a_{2i-1}x^{2i-1} \in R_0[x] \mid a_{2i-1} \in R, n \in N\}$; (iii) $E_0[x;\delta]$, where E is a subring containing all idempotents of R.*

An ideal I of a ring R is called *δ-ideal* whenever $\delta(I) \subseteq I$.

Theorem 2.7. *Let R be a reduced ring. Then the following are equivalent:*

(1) *R is quasi-Baer;*
(2) *$R[x;\delta] \in q\mathcal{B}_{r2}$;*
(3) *$(R[x;\delta], +, .)$ is quasi-Baer;*
(4) *$R_0[x;\delta] \in q\mathcal{B}_{r1}$.*

Proof. (1)⇒(2) Let J be an ideal of $R[x;\delta]$ and $B = r_{R[x;\delta]}(J)$. Let J^1 and B^1 denote the set of all coefficients of elements of J and B respectively. Let $J^{1(\delta)}$ and $B^{1(\delta)}$ be the δ-ideals of R generated by J^1 and B^1 respectively. Hence $r_R(J^{1(\delta)}) = r_R(J^1)$, by Lemma 2.1. We claim that $r_R(J^{1(\delta)}) = B^{1(\delta)}$ and $r_S(J) = B_0^{1(\delta)}[x;\delta]$. Since $0 \in J$, we have $B \subseteq R_0[x;\delta]$. Let $\sum_{i=1}^{n} b_i x^i \in B$ and $(x)g = \sum_{j=0}^{m} g_j x^j \in J$. Then $(\sum_{j=0}^{m} g_j x^j) \circ (\sum_{i=1}^{n} b_i x^i) = 0$ and that $b_i g_j = g_j b_i = 0$ for each $1 \le i \le n, 1 \le j \le m$, since R is reduced. Hence $((x)g + b_i x^{2m+1}) \circ x^2 - b_i x^{2m+1} \circ x^2 = g_0^2 + \cdots + g_0 b_i x^{2m+1} \in J$ for each $1 \le i \le n$. Therefore $b_i g_0 b_i = 0$ for each $1 \le i \le n$, since R is reduced. Hence $gb = bg = 0$ for each $g \in J^1$ and $b \in B^1$. Consequently $g\delta^j(b) = b\delta^j(g) = 0$ for each nonnegative integers j and $b \in B^1$, $g \in J^1$, by Lemma 2.1. Therefore $B^{1(\delta)} \subseteq r_R(J^1) = r_R(J^{1(\delta)})$ and $B_0^{1(\delta)}[x;\delta] \subseteq r_S(J)$. But $r_S(J) = B \subseteq B_0^{1(\delta)}[x;\delta]$, so $r_S(J) = B_0^{1(\delta)}[x;\delta]$. Let $t \in r_R(J^{1(\delta)})$. Then $tJ^1 = J^1 t = 0$ and that $\sum_{j=0}^{m} g_j x^j \circ tx = 0$ for each $\sum_{j=0}^{m} g_j x^j \in J$. Hence $tx \in B$ and that $t \in B^1$. Therefore $r_R(J^{1(\delta)}) = B^{1(\delta)}$. Since R is quasi-Baer and every idempotent of R is central, there exists an idempotent $e \in R$ such that $r_R(J^{1(\delta)}) = eR = Re$. Then

$r_{R[x;\delta]}(J) = eR_0[x;\delta] = ex \circ R_0[x;\delta] = r_{R[x;\delta]}((1-e)x))$, since $\delta(e) = 0$. Therefore $R[x;\delta] \in q\mathcal{B}_{r2}$.

(2)⇒(1) Let I be an ideal of R. Assume $I^{(\delta)}$ be the δ-ideal of R generated by I. Then $I^{(\delta)}[x;\delta]$ is a left nearring of differential polynomials with coefficients from $I^{(\delta)}$. We first show that $I^{(\delta)}[x;\delta]$ is an ideal of $R[x;\delta]$. Let $(x)a = \sum_{i=0}^{n} a_i x^i \in I^{(\delta)}[x;\delta]$ and $(x)f, (x)g = \sum_{j=0}^{m} g_j x^j \in R[x;\delta]$. Observe that $(x)f \circ (x)a = \sum_{i=0}^{\infty} a_i((x)f)^i \in I^{(\delta)}[x;\delta]$ and $((x)a + (x)f) \circ (x)g - (x)f \circ (x)g = \sum_{j=1}^{m} g_j((x)a + (x)f)^j - \sum_{j=1}^{m} g_j((x)f)^j = \sum_{j=1}^{m} g_j[((x)a + (x)f)^j - ((x)f)^j] \in I^{(\delta)}[x;\delta]$, since the coefficients of $[((x)a + (x)f)^j - ((x)f)^j]$ and $a_j((x)f)^j$ belong to $I^{(\delta)}$ for each j. Therefore $I^{(\delta)}[x;\delta]$ is an ideal of $R[x;\delta]$. Since $R[x;\delta] \in q\mathcal{B}_{r2}$, there exists an idempotent $(x)E = e_1x + e_0 \in R[x;\delta]$, where e_1 is an idempotent in R with $e_1e_0 = 0$, such that $r_{R[x;\delta]}(I^{(\delta)}[x;\delta]) = r_{R[x;\delta]}((x)E)$. Since $-e_0+(1-e_1)x \in r_{R[x;\delta]}((x)E)$, we have $e_0 = 0$. On the other hand $r_{R[x;\delta]}(e_1x) = (1-e_1)x \circ R_0[x;\delta] = (1-e_0)R_0[x;\delta]$. One can show that $r_R(I) = (1-e_1)R$. Therefore R is quasi-Baer.

The equivalence of (1) and (3) follows from Hong et al. [15].

(4)⇒(1) Let I be an ideal of R. Assume that $I^{(\delta)}$ be the δ-ideal of R generated by I. Hence $I_0^{(\delta)}[x;\delta]$, the 0-symmetric left nearring of differential polynomials with coefficients from $I^{(\delta)}$, is an ideal of $R_0[x;\delta]$. Since $R_0[x;\delta] \in q\mathcal{B}_{r1}$, there exists an idempotent $(x)\varepsilon \in R_0[x;\delta]$ such that $r_{R_0[x;\delta]}(I_0^{(\delta)}[x;\delta]) = (x)\varepsilon \circ R_0[x;\delta]$. By Lemma 2.2, $(x)\varepsilon = ex$ for some idempotent $e \in R$. Hence $r_{R_0[x;\delta]}(I_0^{(\delta)}[x;\delta]) = (x)\varepsilon \circ R_0[x;\delta] = eR_0[x;\delta]$, since $\delta(e) = 0$ and e is a central idempotent of R. Since $I \subseteq I^{(\delta)}$, hence $reax = ax \circ (ex \circ rx) = 0$ for each $a \in I$ and $r \in R$. Consequently $eRI = IeR = 0$, since R is reduced and e is a central idempotent of R. Hence $eR \subseteq r_R(I)$. Now, let $t \in r_R(I)$. Then $It = tI = 0$ an that $tI^{(\delta)} = 0$, by Lemma 2.1. Hence $I_0^{(\delta)}[x;\delta] \circ tx = 0$. Thus $tx \in r_{S_0}(I_0^{(\delta)}[x;\delta]) = ex \circ R_0[x;\delta]$. Therefore $tx = ex \circ tx = tex$ and that $t = et \in eR$. Consequently $r_R(I) = eR$. Therefore R is a quasi-Baer ring.

(1)⇒(4) Assume that R is a quasi-Baer ring. Let J be an ideal of $R_0[x;\delta]$. Assume that $J^{1(\delta)}$ be the δ-ideal of R generated by the set of all coefficients of elements of J. Then $J_0^{1(\delta)}[x;\delta]$, the 0-symmetric left nearring of differential polynomials with coefficients from $J^{1(\delta)}$, is an ideal of $R_0[x;\delta]$. By using Lemma 2.1 one can show that $r_{R_0[x;\delta]}(J) = r_{R_0[x;\delta]}(J_0^{1(\delta)}[x;\delta])$. Since R is quasi-Baer, hence $\ell_R(J^{1(\delta)}) = r_R(J^{1(\delta)}) = eR$ for some idempotent $e \in R$. We show that $r_{R_0[x;\delta]}(J) = ex \circ R_0[x;\delta]$. Since $e \in r_R(J^{1(\delta)})$, we have $ex \circ R_0[x;\delta] \subseteq r_{R_0[x;\delta]}(J)$. Now, let $(x)g = g_1x+\cdots+g_mx^m \in r_{R_0[x;\delta]}(J) = r_{R_0[x;\delta]}(J_0^{1(\delta)}[x;\delta])$. Then $J^{1(\delta)}g_i = g_iJ^{1(\delta)} = 0$ for each $i = 1, \cdots, m$, since R is reduced. Therefore $g_i \in r_R(J^{1(\delta)}) = eR$

and that $g_i = eg_i = g_ie$ for each $i = 1, \cdots, m$. Hence $(x)g = ex \circ (x)g$, since $\delta(e) = 0$. Consequently $r_{R_0[x;\delta]}(J) = r_{R_0[x;\delta]}(J_0^{1(\delta)}[x;\delta]) = ex \circ R_0[x;\delta]$, which implies $R_0[x;\delta] \in q\mathcal{B}_{r1}$. □

Corollary 2.8. *Let R be a reduced ring. Then the following are equivalent:*

(1) *R is quasi-Baer;*
(2) *$R[x] \in q\mathcal{B}_{r2}$;*
(3) *$(R[x], +, .)$ is quasi-Baer;*
(4) *$R_0[x] \in q\mathcal{B}_{r1}$.*

Acknowledgment. The author would like to thank the referee for the valuable suggestions and comments.

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