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THE STRUCTURE OF $\mathcal{U}(\mathbb{F}_{5^k}D_{20})$

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Abstract. The structure of the unit group of the group algebra of the group D_{20} over any field of characteristic 5 is established in terms of split extensions of cyclic groups.

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1. Introduction

Let KG denote the group algebra KG of the group G over the field K and $\mathcal{U}(KG)$ denote the unit group of KG. The homomorphism $\varepsilon : KG \longrightarrow K$ given by $\varepsilon\left(\sum_{g\in G} a_g g\right) = \sum_{g\in G} a_g$ is called *the augmentation mapping* of KG. It is well known that $\mathcal{U}(KG) = V(KG) \times \mathcal{U}(K)$ where $V(KG)$ is the units of augmentation 1. Currently there exists techniques to find the decomposition of KG and hence the structure of $\mathcal{U}(KG)$ when the the characteristic of the field K does not divide the order of the group G. See [9] for further details on group algebras. However very little is known about $\mathcal{U}(KG)$ when the characteristic of the field K is p and the order of the group is ap^m where p is a prime, $(a,p) = 1$ and $a, m \in \mathbb{N}_0$.

It is well known that if G is a finite p-group and K is a field of characteristic p, then $V(KG)$ is a finite p-group of order $|K|^{|G|-1}$. Let $\mathbb{F}_{p^k}$ is the Galois field of p^k-elements. A basis for $V(\mathbb{F}_pG)$ is determined where $\mathbb{F}_p$ is the Galois field of p elements and G is an abelian p-group in [10] and a basis for $V_*(FG)$ is established where F is any field of characteristic p and G is an abelian p-group in [1] where $V_*(FG)$ are the unitary units of $V(FG)$. Also in [2], there are conditions provided when $V_*(FG)$ is normal in $V(FG)$.

Let D_{2p^m} be the dihedral group of order $2p^m$ where p is a prime and $m \in \mathbb{N}_0$. In [3] and [6], the structure of $\mathcal{U}(\mathbb{F}_{3^k}D_6)$ and $\mathcal{U}(\mathbb{F}_{5^k}D_{10})$ are established in terms of split extensions of elementary abelian groups. The order of $\mathcal{U}(\mathbb{F}_{p^k}D_{2p^m})$ is determined to be $p^{2k(p^m-1)}(p^k-1)^2$ in [5].

Let $J(KG)$ and denote the jacobson radical of KG and $Z(G)$ denote the center of G. In [11], it is shown that $\mathcal{V}_1$ and $\mathcal{V}_1/Z(\mathcal{V}_1)$ are elementary abelian 3-groups where $\mathcal{V}_1 = 1 + J(\mathbb{F}_{3^k}D_6)$. In [8], it is also shown that $\mathcal{U}(\mathbb{F}_{5^k}D_{10})/\mathcal{V}_2 \cong {C_{5^k-1}}^2$, $\mathcal{V}_2$ is nilpotent of class 4 and $Z(\mathcal{V}_2) \cong {C_5}^{3k}$ where $\mathcal{V}_2 = 1 + J(\mathbb{F}_{5^k}D_{10})$ and C_n is the cyclic group of order n. Our main result is:

Theorem 1.1. $V(\mathbb{F}_{5^k}D_{20}) \cong (({C_5}^{10k} \rtimes {C_5}^{4k}) \rtimes {C_5}^{2k}) \rtimes {C_{5^k-1}}^3$.

Define a circulant matrix over R to be

$$\text{circ}(a_1, a_2, \ldots, a_n) = \begin{pmatrix} a_1 & a_2 & a_3 & \cdots & a_n \\ a_n & a_1 & a_2 & \cdots & a_{n-1} \\ a_{n-1} & a_n & a_1 & \cdots & a_{n-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_2 & a_3 & a_4 & \cdots & a_1 \end{pmatrix}$$

where $a_i \in R$. For further details on circulant matrices see Davis [4].

If $G = \{g_1, \ldots, g_n\}$, then denote by $M(G)$ the matrix $({g_i}^{-1}g_j)$ where $i,j = 1, \ldots, n$. Similarly, if $w = \sum_{i=1}^{n} \alpha_{g_i} g_i \in RG$ where R is a ring, then denote by $M(RG, w)$ the matrix $(\alpha_{{g_i}^{-1}g_j})$, which is called the RG-matrix of w.

Theorem 1.2. (*see* [7]) *Let G be a finite group of order n. There is a ring isomorphism between RG and the $n \times n$ G-matrices over R, which is given by $\sigma : w \mapsto M(RG, w)$.*

Let $\kappa = \sum_{i=0}^{4} x^{2i}(\alpha_{i+1} + \alpha_{i+6}x^5 + \alpha_{i+11}y + \alpha_{i+16}x^5 y) \in \mathbb{F}_{5^k}D_{20}$ where $a_i \in \mathbb{F}_{5^k}$, then

$$\sigma(\kappa) = \begin{pmatrix} A & B & C & D \\ B & A & D & C \\ C^T & D^T & A^T & B^T \\ D^T & C^T & B^T & A^T \end{pmatrix}$$

where $A = \text{circ}(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5)$, $B = \text{circ}(\alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha_{10})$,
$C = \text{circ}(\alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{14}, \alpha_{15})$ and $D = \text{circ}(\alpha_{16}, \alpha_{17}, \alpha_{18}, \alpha_{19}, \alpha_{20})$.

Proposition 1.3. (*see* [5]) *Let $A = circ(a_0, a_2, \ldots, a_{p^m-1})$, where $a_i \in \mathbb{F}_{p^k}$, p is a prime and $m \in \mathbb{N}_0$. Then*

$$det(A) = \sum_{i=0}^{p^m-1} {a_i}^{p^m}.$$

Proof of Main Theorem. Define the group epimorphism

$$\theta : \mathcal{U}(\mathbb{F}_{5^k}D_{20}) \longrightarrow \mathcal{U}(\mathbb{F}_{5^k}(C_2 \times C_2)) \text{ given by}$$

$$\sum_{i=0}^{4} x^{2i}(\alpha_{i+1}+\alpha_{i+6}x^5+\alpha_{i+11}y+\alpha_{i+16}x^5y) \longmapsto \sum_{i=0}^{4}(\alpha_{i+1}+\alpha_{i+6}\overline{x}+\alpha_{i+11}\overline{y}+\alpha_{i+16}\overline{x}\,\overline{y})$$

where $C_2 \times C_2 = \langle \overline{x}, \overline{y} \,|\, \overline{x}^2 = \overline{y}^2 = 1\, \overline{xy} = \overline{yx}\rangle$. Let

$$\psi : \mathcal{U}(\mathbb{F}_{5^k}(C_2 \times C_2)) \longrightarrow \mathcal{U}(\mathbb{F}_{5^k}D_{20})$$

be the group homomorphism defined by $a + b\overline{y} + c\overline{x} + d\overline{xy} \mapsto a + bx^5 + cy + dx^5y$. Then $\theta \circ \psi(a+b\overline{y}+c\overline{x}+d\overline{x}\,\overline{y}) = a+b\overline{y}+c\overline{x}+d\overline{x}\,\overline{y}$ and $\mathcal{U}(\mathbb{F}_{5^k}D_{20})$ is a split extension of $\mathcal{U}(\mathbb{F}_{5^k}(C_2 \times C_2))$ by $ker(\theta)$. Thus $\mathcal{U}(\mathbb{F}_{5^k}D_{20}) \cong H \rtimes \mathcal{U}(\mathbb{F}_{5^k}(C_2 \times C_2))$ where $H = ker(\theta)$. Let $\alpha = \sum_{i=0}^{4} x^{2i}(\alpha_{i+1} + \alpha_{i+6}x^5 + \alpha_{i+11}y + \alpha_{i+16}x^5y) \in \mathcal{U}(\mathbb{F}_{5^k}D_{20})$ where $\alpha_i \in \mathbb{F}_{5^k}$. Now $\alpha \in H$ if and only if $\sum_{i=0}^{4} \alpha_{i+1} = 1$ and

$$\sum_{j=0}^{4} \alpha_{j+6} = \sum_{l=0}^{4} \alpha_{l+11} = \sum_{m=0}^{4} \alpha_{m+16} = 0.$$

Thus $|H| = (5^{4k})^4 = 5^{16k}$.

Lemma 1.4. *H has exponent 5.*

Proof. Let

$$h = 1 - \sum_{i=1}^{4}(\alpha_i + \alpha_{i+4}x^5 + \alpha_{i+8}y + \alpha_{i+12}x^5y) + \sum_{i=1}^{4} x^{2i}(\alpha_i + \alpha_{i+4}x^5 + \alpha_{i+8}y + \alpha_{i+12}x^5y) \in H,$$

then

$$(\sigma(\alpha))^5 = \begin{pmatrix} A^5 & 0 & 0 & 0 \\ 0 & A^5 & 0 & 0 \\ 0 & 0 & (A^T)^5 & 0 \\ 0 & 0 & 0 & (A^T)^5 \end{pmatrix}$$

where $A = \text{circ}\left(1 + \sum_{i=1}^{4}(-\alpha_i), \alpha_1, \alpha_2, \alpha_3, \alpha_4\right)$ and $\alpha_i \in \mathbb{F}_{5^k}$. Using Proposition 1,

$$A^5 = \left(\left(1 + \textstyle\sum_{i=1}^{4}(-\alpha_i)\right)^5 + \textstyle\sum_{i=1}^{4}(\alpha_i)^5\right) I_5 = I_5.$$

□

Lemma 1.5. *Let T be the set of elements H of the form $1+r\sum_{i=0}^{4} ix^{2i}y$ where $r\in\mathbb{F}_{5^k}$. Then $T\cong {C_5}^k$.*

Proof. Let $\alpha=1+r\sum_{i=0}^{4} ix^{2i}y\in T$ and $\beta=1+s\sum_{i=0}^{4} ix^{2i}y\in T$ where $r,s\in\mathbb{F}_{5^k}$. Then

$$\alpha\beta=1+(r+s)\sum_{i=0}^{4} ix^{2i}y\in T.$$

Thus T is closed under multiplication. It can easily be shown that T is abelian. □

Lemma 1.6. $|N_H(T)|=5^{14k}$.

Proof. $N_H(T)=\{h\in H\,|\,T^h=T\}$. Let $t=1+r\sum_{i=0}^{4} ix^{2i}y\in T$ and

$$h=1-\sum_{i=1}^{4}(\alpha_i+\alpha_{i+4}x^5+\alpha_{i+8}y+\alpha_{i+12}x^5y)$$
$$+\sum_{i=1}^{4}x^{2i}(\alpha_i+\alpha_{i+4}x^5+\alpha_{i+8}y+\alpha_{i+12}x^5y)\in H,$$

where $\alpha_i,r\in\mathbb{F}_{5^k}$.

$$\sigma(t^h)=\begin{pmatrix} I_5 & 0 & C & D\\ 0 & I_5 & D & C\\ C^T & D & I_5 & 0\\ D & C^T & 0 & I_5\end{pmatrix}$$

where $C=\mathrm{circ}(r\delta_1,r(1+\delta_1),r(2+\delta_1),r(3+\delta_1),r(4+\delta_1))$ and $D=\mathrm{circ}(r\delta_2,r\delta_2,r\delta_2,r\delta_2,r\delta_2)$, $\delta_1=2\sum_{i=1}^{4}i\alpha_i$ and $\delta_2=2\sum_{i=1}^{4}i\alpha_{i+4}$. Then $h\in N_H(T)$ iff $\delta_1=\delta_2=0$. Therefore $\alpha_4=\alpha_1+2\alpha_2+3\alpha_3$ and $\alpha_8=\alpha_5+2\alpha_6+3\alpha_7$. Thus every element of $N_H(T)$ has the form

$$1+\sum_{i=1}^{3}\left[(4-i)\alpha_i+\alpha_ix^{2i}+(i\alpha_i)x^8+(4-i)\alpha_{i+3}x^5+\alpha_{i+3}x^{2i+5}+(i\alpha_{i+3})x^3\right]$$
$$+\sum_{j=1}^{4}\left[(-\alpha_{j+6})y+\alpha_{j+6}x^{2j}y+(-\alpha_{j+10})x^5y+\alpha_{j+10}x^{2j+5}y\right].$$

Therefore $|N_H(T)|=5^{14k}$. □

Lemma 1.7. *Let S be the set of elements of H of the form*

$$1+r(x^2+x^4)(1-x^4)(1+y)+r_1(x+x^3)(x^6-1)(1+y)$$

where $r, r_1 \in \mathbb{F}_{5^k}$. *Then* $S \cong {C_5}^{2k}$.

Proof. Let

$$\alpha = 1 + r(x^2 + x^4)(1 - x^4)(1 + y) + r_1(x + x^3)(x^6 - 1)(1 + y) \in S$$

and

$$\beta = 1 + s(x^2 + x^4)(1 - x^4)(1 + y) + s_1(x + x^3)(x^6 - 1)(1 + y) \in S$$

where $r, r_1, s, s_1 \in \mathbb{F}_{5^k}$. Then

$$\alpha\beta = 1 + (r + s)(x^2 + x^4)(1 - x^4)(1 + y) + (r_1 + s_1)(x + x^3)(x^6 - 1)(1 + y).$$

Thus S is closed under multiplication. It can easily be shown that S is abelian. □

Lemma 1.8. $H \cong N_H(T) \rtimes S$.

Proof. Let

$$\begin{aligned} n = 1 &+ \sum_{i=1}^{3} \left[(4-i)\alpha_i + \alpha_i x^{2i} + (i\alpha_i)x^8 + (4-i)\alpha_{i+3} + \alpha_{i+3}x^{2i+5} + (i\alpha_{i+3})x^3\right] \\ &+ \sum_{j=1}^{4} \left[(-\alpha_{j+6})y + \alpha_{j+6}x^{2j}y + (-\alpha_{j+10})x^5 y + \alpha_{j+10}x^{2j+5}y\right] \in N_H(T) \end{aligned}$$

and $s = 1 + r(x^2 + x^4)(1 - x^4)(1 + y) + r_1(x + x^3)(x^6 - 1)(1 + y) \in S$ where $a_i, r, r_1 \in \mathbb{F}_{5^k}$. Then

$$\sigma(n^s) = \begin{pmatrix} A & B & C & D \\ B & A & D & C \\ C^T & D^T & A^T & B^T \\ D^T & C^T & B^T & A^T \end{pmatrix}$$

where $A = \mathrm{circ}\left[1 + \sum_{i=1}^{3}(4-i)\alpha_i, \alpha_1 + \delta_1, \alpha_2 + 2\delta_1, \alpha_3 + 3\delta_1, \sum_{i=1}^{3} i\alpha_i + 4\delta_1\right]$,
$B = \mathrm{circ}\left[1 + \sum_{i=1}^{3}(4-i)\alpha_{i+3}, \alpha_4 + \delta_2, \alpha_5 + 2\delta_2, \alpha_6 + 3\delta_2, \sum_{i=1}^{3} i\alpha_{i+3} + 4\delta_2\right]$,
$C = \mathrm{circ}\left[\sum_{j=1}^{4}(-\gamma_j), \gamma_1, \gamma_2, \gamma_3, \gamma_4\right]$, $D = \mathrm{circ}\left[\sum_{j=1}^{4}(-\gamma_{j+4}), \gamma_5, \gamma_6, \gamma_7, \gamma_8\right]$,

$$\delta_1 = \sum_{i=1}^{4} irr_1(\alpha_{i+10} + 3i\alpha_{i+6}(r^2 + {r_1}^2)) + r((\alpha_7 - \alpha_8) + (\alpha_{10} - \alpha_9)) + r_1((\alpha_{11} - \alpha_{12}) + (\alpha_{14} - \alpha_{13})),$$

$$\delta_2 = \sum_{i=1}^{4} irr_1(\alpha_{i+6} + 3i\alpha_{i+10}(r^2 + {r_1}^2)) + r((\alpha_{11} - \alpha_{12}) + (\alpha_{14} - \alpha_{13})) + r_1((\alpha_7 - \alpha_8) + (\alpha_{10} - \alpha_9)),$$

$\alpha_i, r, r_1 \in \mathbb{F}_{5^k}$ and the γ_i's are functions of α_i, r and r_1.

Clearly $n^s \in N_H(T)$, S normalizes $N_H(T)$ and $\langle N_H(T), S\rangle = N_H(T)S$. By the Second Isomorphism Theorem $N_H(T)S/S \cong S/N_H(T) \cap S$. Now $N_H(T) \cap S = \{1\}$, therefore $H = N_H(T)S = N_H(T) \rtimes S$. $\square$

Consider the set

$$U = \{\ 1 + \sum_{i=1}^{3} \left[(4-i)\alpha_i + \alpha_i x^{2i} + (i\alpha_i)x^8 + (4-i)\alpha_{i+3} + \alpha_{i+3}x^{2i+5} + (i\alpha_{i+3})x^3\right]$$
$$+ [3(\alpha_7+\alpha_8) + \alpha_7(x^2+x^8) + \alpha_8(x^4+x^6) + 3(\alpha_9+\alpha_{10})x^5 + \alpha_9(x^3+x^7) + \alpha_{10}(x+x^9)]y\ \},$$

where $\alpha_i \in \mathbb{F}_{5^k}$. It can easily be shown that U is a group and $U \cong {C_5}^{10k}$. Also it can be shown that the set

$$V = \{\ 1 + 3(\alpha_1+\alpha_2) + \alpha_1(x^2+x^8) + \alpha_2(x^4+x^8) + 3(\alpha_3+\alpha_4)x^5 + \alpha_3(x^3+x^7) + \alpha_4(x+x^9)$$
$$+ [\alpha_5(1-x^8) + \alpha_6(x^2-x^6) + \alpha_7(x^5-x^3) + \alpha_8(x^7-x)]y\ \},$$

is a group and $V \cong {C_5}^{8k}$ where $\alpha_i \in \mathbb{F}_{5^k}$.

Lemma 1.9. $N_H(T) \cong {C_5}^{10k} \rtimes {C_5}^{4k}$.

Proof. Let

$$u = 1 + \sum_{i=1}^{3} \left[(4-i)\alpha_i + \alpha_i x^{2i} + (i\alpha_i)x^8 + (4-i)\alpha_{i+3} + \alpha_{i+3}x^{2i+5} + (i\alpha_{i+3})x^3\right]$$
$$+ [3(\alpha_7+\alpha_8) + \alpha_7(x^2+x^8) + \alpha_8(x^4+x^6) + 3(\alpha_9+\alpha_{10})x^5 + \alpha_9(x^3+x^7) + \alpha_{10}(x+x^9)]y \in U$$

and

$$v = 1 + 3(\beta_1+\beta_2) + \beta_1(x^2+x^8) + \beta_2(x^4+x^8) + 3(\beta_3+\beta_4)x^5 + \beta_3(x^3+x^7) + \beta_4(x+x^9)$$
$$+ [\beta_5(1-x^8) + \beta_6(x^2-x^6) + \beta_7(x^5-x^3) + \beta_8(x^7-x)]y \in V$$

where $\alpha_i, \beta_j \in \mathbb{F}_{3^k}$. Then

$$\sigma(u^v) = \begin{pmatrix} A & B & C & D \\ B & A & D & C \\ C & D & A^T & B^T \\ D & C & B^T & A^T \end{pmatrix}$$

where $A = \operatorname{circ}\left(1 + \sum_{i=1}^{3}(4-i)\alpha_i, \alpha_1+\delta_1, \alpha_2+2\delta_1, \alpha_3+3\delta_1, \sum_{i=1}^{3} i\alpha_i + 4\delta_1\right)$,
$B = \operatorname{circ}\left(1 + \sum_{i=1}^{3}(4-i)\alpha_{i+3}, \alpha_4+\delta_2, \alpha_5+2\delta_2, \alpha_6+3\delta_2, \sum_{i=1}^{3} i\alpha_{i+3} + 4\delta_2\right)$,
$C = \operatorname{circ}(3\alpha_7+3\alpha_8+\delta_3, \alpha_7+\delta_3, \alpha_8+\delta_3, \alpha_8+\delta_3, \alpha_7+\delta_3)$,
$D = \operatorname{circ}(3\alpha_9+3\alpha_{10}+\delta_4, \alpha_9+\delta_4, \alpha_{10}+\delta_4, \alpha_{10}+\delta_4, \alpha_9+\delta_4)$,

$\delta_1 = (\alpha_7 - \alpha_8)(2\beta_5 + \beta_6) + (\alpha_9 - \alpha_{10})(2\beta_7 + \beta_8)$,
$\delta_2 = (\alpha_7 - \alpha_8)(2\beta_7 + \beta_8) + (\alpha_9 - \alpha_{10})(2\beta_5 + \beta_6)$,
$\delta_3 = (\alpha_2 - \alpha_3)(2\beta_5 + \beta_6) + \gamma_1(\alpha_7 - \alpha_8) + (\alpha_5 - \alpha_6)(2\beta_7 + \beta_8) + \gamma_2(\alpha_9 - \alpha_{10})$,
$\delta_4 = (\alpha_2 - \alpha_3)(2\beta_7 + \beta_8) + \gamma_1(\alpha_9 - \alpha_{10}) + (\alpha_5 - \alpha_6)(2\beta_5 + \beta_6) + \gamma_2(\alpha_7 - \alpha_8)$,
$\gamma_1 = 3{\beta_5}^2 + 3\beta_5\beta_6 + 2{\beta_6}^2 + 3{\beta_7}^2 + 3\beta_7\beta_8 + 2{\beta_8}^2$ and $\gamma_2 = 4\beta_6\beta_8 + \beta_5\beta_7 + 3\beta_6\beta_7 + 3\beta_5\beta_8$.

Clearly $u^v \in U$ and V normalizes U and $\langle U, V\rangle = UV$. Let

$$R = U \cap V = \left\{1 + 3(a+b) + a(x^2 + x^8) + b(x^4 + x^8) + 3(c+d) + c(x^3 + x^7) + d(x + x^9)\right\}$$

where $a, b, c, d \in \mathbb{F}_{3^k}$. Now $N_H(T) = UV$ by the second Isomorphism Theorem. Let $V \cong R \times W \cong {C_5}^{4k} \times {C_5}^{4k}$ since V completely reduces. Clearly $W \cap U = \{1\}$ and W normalizes V. Thus $N_H(T) \cong U \rtimes W \cong {C_5}^{10k} \rtimes {C_5}^{4k}$. □

Recall that $\mathcal{U}(\mathbb{F}_{5^k}D_{20}) \cong H \rtimes \mathcal{U}(\mathbb{F}_{5^k}(C_2 \times C_2))$. Also $H \cong N_H(T) \rtimes S \cong ({C_5}^{10k} \rtimes {C_5}^{4k}) \rtimes {C_5}^{2k}$. Now

$$\mathbb{F}_{5^k}(C_2 \times C_2) \cong (\mathbb{F}_{5^k}C_2)C_2 \cong (\mathbb{F}_{5^k} \oplus \mathbb{F}_{5^k})C_2 \cong \mathbb{F}_{5^k}C_2 \oplus \mathbb{F}_{5^k}C_2 \cong \mathbb{F}_{5^k} \oplus \mathbb{F}_{5^k} \oplus \mathbb{F}_{5^k} \oplus \mathbb{F}_{5^k}.$$

Therefore

$$\begin{aligned}\mathcal{U}(\mathbb{F}_{5^k}D_{20}) &\cong \left(({C_5}^{10k} \rtimes {C_5}^{4k}) \rtimes {C_5}^{2k}\right) \rtimes {C_{5^k-1}}^4 \\ &\cong \left[(({C_5}^{10k} \rtimes {C_5}^{4k}) \rtimes {C_5}^{2k}) \rtimes {C_{5^k-1}}^3\right] \times \mathcal{U}(\mathbb{F}_{5^k}).\end{aligned}$$

□

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